Lecture 10, Sep 29, 2022

3-Variable K-Maps

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

- Group sizes are 1, 2, 4, 8
- Note the inputs are not enumerated in ascending order
  - This is a *grey code*, a sequence of bits where when transitioning between consecutive terms, only 1 bit changes
  - This ensures that adjacent entries in the table only differ by 1 input bit
- The way the table is arranged shouldn't matter for the final result, as long as minterms are mapped properly
- Example: $f = \sum m(3,7) = \bar{x}_1x_2x_3 + x_1x_2x_3 = x_2x_3(\bar{x}_1 + x_1) = x_2x_3$
  - In the K-Map these terms are adjacent, in the two where x_3 and x_2 are 1, and the value of x_1 does not matter, so this gives us x_2x_3 both non-inverted and no x_1
- Example: minterms m_2, m_6, m_3, m_7 simplifies to x_2
  - Notice that a product term that covers more adjacent cells is cheaper!
- The K-Map also wraps around, e.g. we can combine m_0, m_4 to $\bar{x}_2\bar{x}_3$

Terminology

- Implicant: for a function f, an *implicant* is any product term covered/included by f
  - Can be a simplified or unsimplified term
- Prime Implicant: an implicant for which it is impossible to remove any literal and still have a valid implicant
- Cover: any set of implicants that includes all minterms of a function (every 1 in a K-Map needs to be covered)
- Example: $f(x_1, x_2, x_3) = \sum m(1,4,5,6)$
  - Prime implicants are $x_1\bar{x}_3, x_1\bar{x}_2, \bar{x}_2x_3$
  - Minimal cost cover is $x_1\bar{x}_3 + \bar{x}_2x_3$ (notice $x_1\bar{x}_2$ is not included)

x_3 \ x_1x_2	00	01	11	10
0	0	0	1	1
1	1	0	0	1

- An essential prime implicant is a prime implicant that covers at least one minterm that is not covered by any other prime implicant
  - In the last example $x_1\bar{x}_2$ is not an essential prime implicant
- A minimal cost cover includes all essential prime implicants

4-Variable K-Maps

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8

x_3x_4 \ x_1x_2	00	01	11	10
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

- Group sizes are 1, 2, 4, 8, 16
- Example: $f(x_1, x_2, x_3, x_4) = \sum m(2, 4, 5, 8, 10, 11, 12, 13, 15)$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	1	1
10	1	0	0	1

- Prime implicants: $x_2\bar{x}_3, x_1\bar{x}_3\bar{x}_4, x_1x_2x_4, x_1x_3x_4, x_1\bar{x}_2x_3, \bar{x}_2x_3\bar{x}_4, x_1\bar{x}_2\bar{x}_4$
  - Note minterms 12 and 13 don't form a PI since it's completely inside the PI for minterms 4, 5, 12, 13
- Essential PIs: $x_2\bar{x}_3, \bar{x}_2x_3\bar{x}_4$
- Minimal cost cover: $f = x_2\bar{x}_3 + \bar{x}_2x_3\bar{x}_4 + x_2x_3x_4 + \begin{cases} x_1\bar{x}_3\bar{x}_4 \\ x_2\bar{x}_3\bar{x}_4 \end{cases}$
  - This shows that there can be multiple minimal cost covers