Lecture 6, Sep 20, 2022

Control Volumes/Open Systems

- For these systems we need to take into account the mass flowing in/out of a system and the energy the mass carries
- *Steady flow processes* are devices like pumps, compressors, turbines, etc

Mass Balance

- Consider a generic control volume with an inlet and an outlet
  - Fluid coming in with velocity $\boldsymbol{v}$, with cross section A, entering with length $\mathrm{d}x$, mass δm
    * Note δm is used since mass is crossing the boundary, so it's not a property anymore
    * Length of fluid element inside control volume in time $\mathrm{d}t$ is $\mathrm{d}x = \boldsymbol{v}\,\mathrm{d}t$
    * Volume of fluid element is $A\,\mathrm{d}x = A\boldsymbol{v}\,\mathrm{d}t$
    * Mass of fluid element is $\delta m = \rho A\boldsymbol{v}\,\mathrm{d}t$
  - Mass flow rate $\dot{m} = \dfrac{\delta m}{\mathrm{d}t} = \rho A\boldsymbol{v}$
- Consider a turbine with the fluid expanding as it goes through
  - Pressure goes down as fluid passes through
  - $\dot{m} = \rho A\boldsymbol{v} = \text{const}$
  - Therefore A must be increased to keep $\boldsymbol{v}$ approximately constant, so the flow is smooth
    * Cross section area must be increased to maintain flow rate as gas loses pressure
  - $\dot{m} = \rho A\boldsymbol{v} = \dfrac{A\boldsymbol{v}}{v}$
    * Density $\rho = \dfrac{m}{V}$, which is simply $\dfrac{1}{v}$ where v is the specific volume
    * For an ideal gas $Pv = RT \implies \dfrac{1}{v} = \dfrac{P}{RT}$ so $\dot{m} = \dfrac{PA\boldsymbol{v}}{RT}$
  - At steady state $\dot{m}_{in} = \dot{m}_{out}$
- First law for a control volume:
  - Assume $\dot{m}$ coming in, with $\boldsymbol{v}_1, h_1$, height z_1, outlet has $\boldsymbol{v}_2, h_2, z_2$
  - The system is heated with $\dot{Q}$, and heat is transferred in with $\dot{W}$
    * The work in this case is usually *shaft work* (more on this later)
  - Energy can be transferred with the flow in different ways:
    * Kinetic energy: $ke = \dfrac{1}{2}\boldsymbol{v}^2$
    * Potential energy: $pe = gz$
    * Internal energy: u
    * Flow work: Pv
    * The last two is combined into enthalpy $h = u + Pv$
  - Flow energy per unit mass is $h + ke + Pe = h + \dfrac{\boldsymbol{v}^2}{2} + gz$
  - Energy balance at steady state requires $\dot{E}_{in} = \dot{E}_{out} \implies \dot{Q} + \dot{W} + \dot{m}\left(h_1 + \dfrac{\boldsymbol{v}_1^2}{2} + gz_1\right) = \dot{m}\left(h_2 + \dfrac{\boldsymbol{v}_2^2}{2} + gz_2\right)$
  - $\dot{Q} + \dot{W} = \dot{m}\left((h_2 - h_1) + \dfrac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2} + g(z_2 - z_1)\right)$
    * Define $q = \dfrac{\dot{Q}}{\dot{m}}, w = \dfrac{\dot{W}}{\dot{m}}$
    * $q + w = (h_2 - h_1) + \dfrac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2} + g(z_2 - z_1)$

Definition

The first law for a control volume:

$$\dot{Q} + \dot{W} = \dot{m}\left((h_2 - h_1) + \frac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2} + g(z_2 - z_1)\right)$$

or mass normalized:

$$\dot{q} + \dot{w} = (h_2 - h_1) + \frac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2} + g(z_2 - z_1)$$

Steady Flow Devices

- A turbine thermodynamically is h_1 going in, h_2 coming out, $\dot{W}_{shaft}$ extracted
- A compressor thermodynamically is just a turbine running backwards
- Consider a turbine with $\dot{Q} = 0$, i.e. heat loss is negligible, and changes in KE and PE are negligible:
  - $\dot{Q} + \dot{W} = \dot{m}\left((h_2 - h_1) + \frac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2} + g(z_2 - z_1)\right) \iff \dot{W} = \dot{m}(h_2 - h_1)$
  - For a turbine, T and P at the inlet are high, so $h_1 > h_2$, and $\dot{W} < 0$ (i.e. work is done on the surroundings)
  - Reverse this for a compressor ($h_2 > h_1$), and work is done by the surroundings to the system
- For a turbine/compressor, the work is in the difference of enthalpies
- Example: A gas turbine is supplied 10kg/s of air at $800°\text{C}, 600\text{kPa}$, which leaves at $300°\text{C}, 100\text{kPa}$; what is the power output?
  - Assume ideal gas, then $\Delta h = c_p \Delta T$
  - $\dot{W} = \dot{m}c_p(T_2 - T_1)$
  - $T_{avg} \approx 800\text{K}$, look up c_p
  - $\dot{w} = 10\text{kg/s} \cdot 1.099\text{kJ/kg}°\text{C} \cdot (300 - 800) = -5495\text{kW}$
- For a pump, the goal is usually to change the PE
  - Consider a height change Δz between the inlet and outlet
  - P_1 coming in, P_2 coming out, $\dot{W}$ driving the pump
  - Assume change in KE is negligible, $\dot{Q}$ negligible
    - For an incompressible fluid, density is constant, so as long as the inlet and outlet pipe diameters are the same, KE is the same
  - $\dot{W} = \dot{m}\left((h_2 - h_1) + g(z_2 - z_1)\right)$
  - For an incompressible fluid, then $h_2 - h_1 = c(T_2 - T_1) + v(P_2 - P_1)$ (we can change enthalpy by change in temperature or pressure)
  - Assume no change in temperature so $h_2 - h_1 = v(P_2 - P_1) \implies \dot{W} = \dot{m}(v(P_2 - P_1) + g(z_2 - z_1))$ (i.e. frictional losses are negligible)
    - The work put in is taken out as either a change in pressure or a change in PE
    - If the difference in height between the inlet/outlet is small, then the pressure change is large
- Nozzles and diffusers change the KE (nozzles increase KE by having the inlet larger than the outlet, diffusers decrease KE by having a nozzle backwards)
  - Assume PE change is 0, no heat or work going in
  - $0 = \dot{m}\left((h_2 - h_1) + \frac{\boldsymbol{v}_2^2 - \boldsymbol{v}_1^2}{2}\right)$
  - Assume $\boldsymbol{v}_2 \gg \boldsymbol{v}_1$, so we can neglect $\boldsymbol{v}_1^2$
  - $\boldsymbol{v}_2 = \sqrt{2(h_1 - h_2)}$
  - Enthalpy is converted into KE by increasing velocity
    - Pressure decreases