Lecture 35, Dec 6, 2022

Radiation Analysis

- Consider 2 surfaces with A_1, T_1 and A_2, T_2, both blackbodies
  - Energy from 1 to 2 is $A_1 E_{b1}(T_1) F_{12}$; energy from 2 to 1 is $A_2 E_{b2}(T_2) F_{21}$
  - The net radiative exchange would be $\dot{Q}_{12} = A_1 E_{b1}(T_1) F_{12} - A_2 E_{b2}(T_2) F_{21}$
  - Using reciprocity $A_1 F_{12} = A_2 F_{21}$ so $Q_{12} = A_1 F_{12}(E_{b1} - E_{b2}) = A_1 F_{12} \sigma(T_1^4 - T_2^4)$
- Note for a small body in an enclosure $F_{12} = 1$ which makes our net radiative heat transfer $\dot{Q}_{12} = A_1 \sigma(T_1^4 - T_2^4)$
- For a more realistic analysis, assume an isothermal, opaque, diffuse (ε independent of direction), and gray (ε independent of λ) surface
  - Together this gives us ε constant for a material, which is a fair assumption over a small range
  - J is the radiosity, the total radiative energy that leaves a surface per unit area per unit time
  - When incident radiation G hits the surface we have ρG being reflected, αG being absorbed and εE_b being radiated back
  - $J = \rho G + \varepsilon E_b = \varepsilon E_b + (1 - \varepsilon) G \implies G = \dfrac{J - \varepsilon E_b}{1 - \varepsilon}$
  - The net energy leaving the surface per unit area is $\dfrac{\dot{Q}}{A} = J - G \implies \dot{Q} = A(J - G) = A\left(J - \dfrac{J - \varepsilon E_b}{1 - \varepsilon}\right)$
  - $\dot{Q} = A\dfrac{\varepsilon}{1 - \varepsilon}(E_b - J) = \dfrac{E_b - J}{\frac{1-\varepsilon}{\varepsilon A}}$
  - We can think of $E_b - J$ as the driving force of radiative heat exchange, using a resistive approach $R = \dfrac{1 - \varepsilon}{\varepsilon A}$, known as the surface resistance
    - Note if we had a blackbody then $R = 0$ and so $E_b = J$ for a blackbody
- Consider 2 gray surfaces i and j with radiosities J_i, J_j
  - The radiation from i that reaches j is $J_i A_i F_{ij}$; from j to i is $J_j A_j F_{ji}$
  - $\dot{Q}_{ij} = J_i A_i F_{ij} - J_j A_j F_{ji} = A_i F_{ij}(J_i - J_j)$ by reciprocity
  - $\dot{Q}_{ij} = \dfrac{J_i - J_j}{(A_i F_{ij})^{-1}}$
    - $\dfrac{1}{A_i F_{ij}}$ is known as the space resistance
  - The total resistance combines the surface resistance for i, $\dfrac{1 - \varepsilon_i}{A_i \varepsilon_i}$, the space resistance $\dfrac{1}{A_i F_{ij}}$, and the surface resistance for j, $\dfrac{1 - \varepsilon_j}{A_j \varepsilon_j}$, with the driving force being blackbody radiation on both sides
- $\dot{Q}_{ij} = \dfrac{E_{bi} - E_{bj}}{R_{tot}} = \dfrac{\sigma(T_i^4 - T_j^4)}{\frac{1-\varepsilon_i}{A_i \varepsilon_i} + \frac{1}{A_i F_{ij}} + \frac{1-\varepsilon_j}{A_j \varepsilon_j}}$
- Example: Consider 2 large parallel plates, $T_1 = 1000\text{K}, \varepsilon_1 = 1, T_2 = 500\text{K}, \varepsilon_2 = 0.8$ with equal area; what is $\dfrac{\dot{Q}_{12}}{A}$?
  - $F_{12} = F_{21} = 1$
  - $\dot{Q}_{12} = \dfrac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{A\varepsilon_2}}$
  - $\dfrac{\dot{Q}_{12}}{A} = \dfrac{\sigma(T_1^4 - T_2^4)}{0 + 1 + \frac{1-\varepsilon_2}{\varepsilon_2}} = \varepsilon_2 \sigma(T_1^4 - T_2^4)$
  - Plugging in values we get 45.5kW/m^2
- Example: An finite system with a groove at 40 degrees, $1000\text{K}, \varepsilon = 0.6, 10\text{mm}$ in the middle; what is the radiation heat flux leaving the groove?
  - Isolate the groove, and create an imaginary surface at the top to enclose the surface; this surface would have 0K and $\varepsilon = 1$ (since $\alpha = 1$); call this surface 2

  - We want $\dot{Q}_{12}$ which would be the amount of radiation escaping the groove
  - Start again with $\dot{Q}_{12} = \dfrac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = \dfrac{\sigma T_1^4}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}}}$
  - To find F_{12} we can use reciprocity, and we know $F_{21} = 1$ which gives us $F_{12} = \dfrac{A_2}{A_1} = \sin(20°)$ using geometry
  - Therefore $\dfrac{\dot{Q}_{12}}{A_2} = \dfrac{\sigma T_1^4}{\frac{1-\varepsilon_1}{\varepsilon_1}\frac{A_2}{A_1} + 1}$
  - Plugging in values we get 46.2kW/m^2
- If we have multiple surfaces, we still have a single surface resistance, but we have multiple space resistances for multiple sources
  - $\dot{Q}_1 = \displaystyle\sum_{i=1}^{N} \dot{Q}_{1i}$
  - Due to conservation of energy $\dfrac{E_{b1} - J_1}{R_1} = \displaystyle\sum_{i=1}^{N} \dfrac{J_1 - J_i}{R_{1i}}$
- Note even when $F_{11} > 0$, we still have $\dot{Q}_{11} = 0$ since we assume an isothermal surface
- In a system with multiple surfaces we get a system of resistances, which we can solve by assessing each node and noting that the heat in equals the heat out for all intermediate nodes

Summary

The total radiative heat transfer between two surfaces i and j is given by

$$\dot{Q}_{ij} = \frac{E_{bi} - E_{bj}}{R_{tot}} = \frac{\sigma(T_i^4 - T_j^4)}{\frac{1-\varepsilon_i}{A_i\varepsilon_i} + \frac{1}{A_iF_{ij}} + \frac{1-\varepsilon_j}{A_j\varepsilon_j}}$$

in which $\dfrac{1-\varepsilon}{A\varepsilon}$ terms are the surface resistances, $\dfrac{1}{A_iF_{ij}}$ is the space resistance, and the driving force is the difference in blackbody radiation between the surfaces; for a system with multiple surfaces, each surface has its own surface resistance, and each pair of surfaces has a space resistance between them