Lecture 34, Dec 5, 2022

Kirchhoff's Law

- At a specific temperature, $\varepsilon = \alpha$ (emissivity equals absorptivity)
  - This works if both the source of the radiation and the absorber are at the same temperature
- Consider a large enclosure and a small body inside with surface area A, emissivity ε and absorptivity α, both at temperature T
  - Assume the large isothermal cavity acts as a blackbody
  - At thermal equilibrium T is the same for both the small body and enclosure
  - The radiation on the small body per unit area is $G = \sigma T^4$ so $E_{abs} = \alpha G A = \alpha \sigma T^4 A$
  - The emitted radiation is $E_{emit} = \varepsilon \sigma T^4 A$
  - At equilibrium these must be equal, so $E_{abs} = E_{emit}$ and therefore $\varepsilon = \alpha$
- Note this is only true if temperatures for both radiation sources are the same
  - We can use this assumption if the temperatures are similar but not quite equal
  - e.g. 350K vs. 300K is okay for this assumption, but for solar radiation of 5000K vs 300K for a room temperature object this assumption would not apply

View Factors

- The amount of radiation incident on a surface depends on orientation
- Consider two surfaces, i and j, then F_{ij} or $F_{i \to j}$ is the fraction of radiation leaving the surface i that reaches j directly; this is known as a *view factor*
  - Radiation that reaches the other surface via one or more reflections is not counted
  - Note F_{ij} would be from i to j and F_{ji} is from j to i, and they may not be equal
    - Consider the case of surface 1 completely enclosed by 2; F_{11} would be 0 and F_{12} would be 1 (since the surface is completely enclosed), but F_{21} is not necessarily 1, because F_{22} is nonzero
  - F_{11} would be the fraction of energy leaving 1 that reaches 1
    - In the case of a flat or convex surface this would clearly be 0, but if we have a concave surface, this can be nonzero
- The calculation of view factors is done by integration over the shape
  - View factors can be found in tables
  - F_{ij} are functions of dimensions, distance, and orientation
  - Values are tabulated for 3D and 2D (infinitely going into screen) cases

Analyzing Radiation Heat Transfer

- Consider an enclosure with N surfaces
  - Since the region is completely enclosed $\sum_{j=1}^{N} F_{ij} = 1$, i.e. all radiation leaving i must hit a surface in the enclosure (summation rule)
- If the problem is not enclosed, we can make an imaginary enclosure by creating a surface out of an opening, containing $\alpha, \varepsilon, \rho$ of the opening
  - We can usually assume that it's absorbing everything, with $T = 0$ and $\alpha = 1$
- For every pair of surfaces we have a view factor, so in total we have a matrix of view factors where $M_{ij} = F_{ij}$
- Note not all F_{ij} are independent; the actual number of independent view factors is $\frac{N(N-1)}{2}$, based on the summation rule
- Reciprocity rule: $A_i F_{ij} = A_j F_{ji}$ for any pair of surfaces
  - Consider 2 surfaces 1 and 2, which are blackbodies at $T_1 = T_2$ and $\alpha = \varepsilon = 1$
  - Energy leaving 1 and reaching 2 is $E_{b1}(T_1) A_1 F_{12}$; energy leaving 2 and reaching 1 is $E_{b2}(T_2) A_2 F_{21}$
  - The net energy exchange is $\dot{Q}_{12} = E_{b1}(T_1) A_1 F_{12} - E_{b2}(T_2) A_2 F_{21}$
  - If we have the same temperature and thus thermal equilibrium, then $\dot{Q}_{12} = 0$ and $E_{b1} = E_{b2}$

- This gives us $A_1 F_{12} = A_2 F_{21}$ for a special case, but all of these are geometric parameters independent of T and ε, so this is true in general
- This means that when two areas are equal, the view factors in both directions are equal
- Superposition: We can break up a surface, and its view factor will be the sum of the view factors of the pieces
- Symmetry: if we have an axis of symmetry then view factors are symmetrical
- Example: Imagine a small sphere with a concentric hemisphere, with $A_2 = 2A_1$; find F_{12} and F_{21}
  - Create a third imaginary surface that closes the hemisphere, call it surface 3
  - We know F_{11} is 0 since it's convex
  - Using the plane of symmetry $F_{12} = F_{13} = 0.5$
  - Using reciprocity $F_{21} = \frac{A_1}{A_2} F_{12}$ so $F_{21} = 0.25$

Summary

To find the view factors in a system, use the 3 rules:

1. Summation rule: $\sum_{j=1}^{N} F_{ij} = 1$ (sum of all outgoing view factors from a surface is 1)
2. Reciprocity rule: $A_i F_{ij} = A_j F_{ji}$ for any pair of surfaces
3. F_{ii} is 0 for any convex surface