Lecture 32, Nov 29, 2022

Forced Convection

- Consider a plate with a boundary layer forming, with laminar, transitional, and turbulent zones, with height $\delta_v(x)$
  - The boundary layer becomes turbulent as shear stress goes down due to the velocity gradient decreasing as the boundary layer gets thicker
- Recall $\tau = C_F \frac{\rho v_\infty^2}{2}$
  - C_F, the friction coefficient, has a local value depending on where on the shape you are
  - In practice friction goes down in the laminar regime, goes up in the transition, and then goes down again the turbulent regime
    - * This is again due to the growth of the boundary layer
  - For laminar flow $C_{F,x} = \frac{0.664}{\mathrm{Re}_x^{\frac{1}{2}}}$
    - * We can average this over the length of the plate: $C_F = \frac{1}{L}\int_0^L \frac{0.664}{\mathrm{Re}_x^{\frac{1}{2}}}\,\mathrm{d}x = \frac{1.328}{\mathrm{Re}_L^{\frac{1}{2}}}$
  - For turbulent flow ($\mathrm{Re}_x > 5 \times 10^5$): $C_{F,x} = \frac{0.0592}{\mathrm{Re}_x^{\frac{1}{2}}}$
    - * This gives an average $C_F = \frac{0.074}{\mathrm{Re}_L^{\frac{1}{5}}}$
- Now consider the thermal boundary layer
  - $h = -\frac{k}{T_s - T_\infty}\left.\frac{\mathrm{d}T}{\mathrm{d}y}\right|_{y=0}$
  - $\frac{\mathrm{d}T}{\mathrm{d}y} \sim \frac{T_s - T_\infty}{\delta_T}$
  - As the thermal boundary layer thickness goes up, the convective heat transfer goes down
  - This gives a local Nousselt number $\mathrm{Nu}_x = \frac{h_x x}{k}$
    - * For laminar flow: $\mathrm{Nu}_x = 0.332\mathrm{Re}^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$ for $\mathrm{Pr} \geq 0.6$
      - This gives an average of $\mathrm{Nu} = 0.664\mathrm{Re}_L^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$
    - * For turbulent flow: $\mathrm{Nu}_x = 0.0296\mathrm{Re}_x^{\frac{4}{5}}\mathrm{Pr}^{\frac{1}{3}}$ for $0.6 \leq \mathrm{Pr} \leq 60, 5 \times 10^5 \leq \mathrm{Re}_x \leq 1 \times 10^7$
      - This gives an average $\mathrm{Nu} = 0.037\mathrm{Re}_L^{\frac{4}{5}}\mathrm{Pr}^{\frac{1}{3}}$
- The film temperature is defined as the average of the surface and free temperatures $T_F = \frac{T_s + T_\infty}{2}$
  - This is to find the average fluid properties

Flows Over Cylinders and Spheres

- Behind a cylinder/sphere there is a turbulent wake, making the boundary layer behave erratically (flow separation, technically not turbulence)
  - Turbulence can still occur, if $\mathrm{Re} = \frac{vD}{\nu} > 2 \times 10^5$
- For a cylinder $\mathrm{Nu} = C\mathrm{Re}^m\mathrm{Pr}^n$ just like for a plate
- The Churchill and Bernstein correlation is more accurate and applies over a broader range
  - $\mathrm{Nu} = 0.3 + \frac{0.62\mathrm{Re}^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}}{\left(1 + \left(\frac{0.4}{\mathrm{Pr}}\right)^{\frac{2}{3}}\right)^{\frac{1}{4}}}\left(1 + \left(\frac{\mathrm{Re}}{28200}\right)^{\frac{5}{8}}\right)^{\frac{4}{5}}$, applicable over $\mathrm{RePr} > 0.2$
- Flow over a sphere is similar: $\mathrm{Nu} = 2 + \left(0.4\mathrm{Re}^{\frac{1}{2}} + 0.06\mathrm{Re}^{\frac{2}{3}}\right)\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{\frac{1}{4}}$, valid for $3.5 \leq \mathrm{Re} \leq 80000, 0.7 \leq \mathrm{Pr} \leq 380$
  - Note all properties are evaluated at T_∞, not T_p like the others (except μ_s)

Summary

For laminar flow over a flat plate (average):

$$\mathrm{Nu} = 0.664\mathrm{Re}_L^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}, \mathrm{Pr} \geq 0.6$$

For turbulent flow over a flat plate (average):

$$\mathrm{Nu} = 0.037\mathrm{Re}_L^{\frac{4}{5}}\mathrm{Pr}^{\frac{1}{3}}, 0.6 \leq \mathrm{Pr} \leq 60, 5 \times 10^5 \leq \mathrm{Re} \leq 1 \times 10^7$$

where Re is the Reynolds number evaluated for the entire plate; material properties and Pr can be determined through the film temperature

$$T_F = \frac{T_s + T_\infty}{2}$$