Lecture 31, Nov 28, 2022

Nusselt Number

- In a turbulent flow we typically have greater heat transfer and shear stress
- The transition point from laminar to turbulent depends on the Reynolds number, the ratio of inertia to viscosity in the fluid
  - For us the characteristic length used is the x position along the plate
- For every geometry, there is a critical Reynolds number at which the transition happens
  - For a flat plate this is about 5×10^5
- We have 2 boundary layers, the velocity boundary layer and the temperature boundary layer; the size of one may be smaller or larger than the other, depending on fluid properties
  - Fluids with high kinematic viscosity (momentum diffusivity, e.g. oils) have thick velocity boundary layers
  - Fluids with high thermal diffusivity ($\alpha = \frac{k}{\rho c}$) have thick thermal boundary layers
- The ratio of the boundary layers is described by the ratio of diffusivities $\frac{\nu}{\alpha}$
- Define the Prandtl number $\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho c}} = \frac{\mu c}{k}$
  - For $\mathrm{Pr} \ll 1$ (e.g. liquid metals), the thermal boundary layer is larger than the velocity boundary layer
  - For $\mathrm{Pr} \gg 1$ (e.g. oils), the velocity boundary layer is larger than the thermal boundary layer
  - For $\mathrm{Pr} \approx 1$ (e.g. gases), the boundary layers are comparable in size
- We can non-dimensionalize h
  - Convective heat transfer scales with D^2
  - If the fluid is not moving, we just have conduction, which scales with kD
  - How much is heat transfer enhanced by the fluid motion?
  - Taking the ratio of these we get $\frac{hD}{k}$
- Define the Nousselt number $\mathrm{Nu} = \frac{hL_c}{k}$ where L_c is a characteristic length, and k is thermal conductivity of the fluid
  - Looks similar to the Biot number, but the thermal conductivity here is of the fluid
- $\mathrm{Nu} = f(\mathrm{Re}, \mathrm{Pr})$ and geometry, and this relationship can be determined experimentally
  - Typically $\mathrm{Nu} = C_0 \mathrm{Re}^m \mathrm{Pr}^n$, with C_0, m, n determined for different geometries

Summary

Typically the convective heat transfer coefficient can be found by

$$\mathrm{Nu} = C_0 \mathrm{Re}^m \mathrm{Pr}^n$$

where $\mathrm{Nu} = \frac{hL_c}{k}$, $\mathrm{Re} = \frac{\rho v L_c}{\mu} = \frac{\nu L_c}{\mu}$ and $\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu c}{k}$ with C_0, m, n determined for different geometries