Lecture 30, Nov 24, 2022

Forced Convection

- Force convection is convection in which the fluid is imparted by external means, as opposed to natural convection which relies on natural fluid motion caused by density changes from temperature
- Up until now we've been using $\dot{Q} = hA(T_i - T_\infty)$ with h already given
  - h could be measured experimentally in a complex system
  - Can we determine h based on system properties?
- Relevant properties:
  - Viscosity μ
  - Density ρ
  - Thermal conductivity k
  - Heat capacity c_p
  - Fluid velocity v_∞ (for a free stream, outside the boundary layer)
  - Shape and size:
    * Characteristic length (length of a plate, diameter of cylinder/sphere)
  - Type of flow (laminar vs turbulent)
- We need to consider the boundary layer, where the fluid slows down near the plate due to the no-slip condition at the boundary
  - We consider this as the region where $v < 0.99v_\infty$
- At the interface between fluid and solid, heat transfer occurs only by conduction
  - $\dot{q} = -k\left.\dfrac{\partial T}{\partial y}\right|_{y=0}$
  - $\dot{q} = h(T_s - T_\infty)$ by Newton's law of cooling
  - Therefore $h(T_s - T_\infty) = -k\left.\dfrac{\partial T}{\partial y}\right|_{y=0} \implies h = \dfrac{-k\left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$
  - Now the problem becomes solving for $\dfrac{\partial T}{\partial y}$, but typically we don't know this
- Consider the boundary layer and assume $T_s > T_\infty$
  - We have both a velocity and a thermal boundary layer; we basically want the slope of this thermal boundary layer
  - Overall h changes with position as local $\dfrac{\partial T}{\partial y}$ at the surface changes, as the boundary layer develops (boundary layer gets thicker as the fluid flows further down the surface)
  - Define a local heat transfer coefficient x where x is along the surface, then the overall h is $\bar{h} = \dfrac{1}{L}\displaystyle\int_0^L h(x)\,\mathrm{d}x$

Boundary Layer Flow

- Typically the boundary layer begins with laminar flow, then becomes turbulent as you go down the surface, with a transitional region in the middle
- The fluid exerts a stress on the plate, $\tau = \mu \left.\dfrac{\partial v}{\partial y}\right|_{y=0}$
- We need to define a "friction coefficient" for the fluid
  - By conservation of energy $\Delta(p\dot{v}) + \Delta\left(\dfrac{\dot{m}v^2}{2}\right) = 0$
  - $p - p_\infty + \dfrac{\rho v^2}{2} - \dfrac{\rho v_\infty^2}{2} = 0 \implies p - p_\infty = \dfrac{\rho v_\infty^2}{2}$
    * This $p - p_\infty$ is the force per unit area felt by the body
    * $\dfrac{\rho v_\infty^2}{2}$ is the inertial force
  - $\dfrac{F}{A} = \tau = c_F\dfrac{\rho v_\infty^2}{2}$ where c_F is the friction coefficient for the fluid, defined for different shapes,

generally about 1
    * For a sphere it's about 0.47, for a plate about 1.17, for a convex hull 2.3, for an airfoil 0.04

Diffusivities

- In molecular diffusion (mass transfer) we have Fick's law $J_A = -D_{AB}\frac{\mathrm{d}C_A}{\mathrm{d}x}$
  - D_{AB} is the mutual diffusion constant for A into solid B with units of m^2/s
  - C_A is the concentration of A
- In heat transfer we have Fourier's law $\dot{q} = -k\frac{\mathrm{d}T}{\mathrm{d}x} \implies \dot{q} = \frac{k}{\rho c_p}\frac{\mathrm{d}}{\mathrm{d}x}(\rho c_p T) = -\alpha\frac{\mathrm{d}}{\mathrm{d}x}(\rho c_p T)$
  - α is the thermal diffusivity
  - $\rho c_p T$ can be thought of as a "concentration of thermal energy", the amount of thermal energy per unit volume with units of J/m^3
- In a fluid μ is the dynamic viscosity; define $\frac{\mu}{\rho} = \nu$ to be the kinematic viscosity (aka momentum diffusivity)
  - Shear stress is in general $\tau = \mu\frac{\mathrm{d}v}{\mathrm{d}y} = \frac{\mu}{\rho}\frac{\mathrm{d}}{\mathrm{d}y}(\rho v) = \nu\frac{\mathrm{d}}{\mathrm{d}y}(\rho v)$
  - This is the diffusion equation again
  - τ can be thought of as a "momentum flux"
  - ρv is the "concentration of momentum", momentum per unit volume