Lecture 3, Sep 13, 2022

Energy

- Energy can be macroscopic (changes in velocity or position, e.g. potential energy, kinetic energy) or microscopic (changes in temperature or pressure, e.g. heating up a gas, compressing a gas)
  - We will be talking about the microscopic forms today

Definition

Internal energy U are all microscopic (molecular) forms of energy stored in a system (e.g. "thermal energy", "chemical energy", etc)

- The total energy of a system is $E = KE + PE + U$
  - Kinetic, potential, and internal energy are the only ways energy can be stored in a system
  - E is an extensive property (it depends on the mass of the system)

Definition

Extensive properties depend on the mass of the system (e.g. energy, volume); intensive properties do not depend on the mass of the system (e.g. temperature, pressure)

- For any extensive property, we can define an intensive property by dividing by the mass
  - e.g. the specific volume $v = \frac{V}{m}$, the specific energy $u = \frac{U}{m}$

Ideal Gas Model

- Assumptions of the model:
  - Hard spheres moving randomly
  - All with the same mass m_e
  - Collisions are elastic
  - No long range interactions, i.e. the only time the molecules interact is when they collide
    * Intermolecular forces within most gases are negligible since the molecules are so far apart
  - Molecules are point masses (no rotational KE, etc)
    * Good approximation for monoatomic gases and noble gases
- Consider a box, a cube of side L, and a molecule with velocity c hitting a wall and bouncing off
  - $\Delta p_x = (-m_e c_x) - (m_e c_x) = -2m_e c_x$
  - With the change in momentum we can find the force
  - The distance travelled between successive collisions is $2L$ as the particle bounces off the opposite wall and comes back, so time between collisions is $\frac{2L}{c_x}$
  - The force on the wall is then $F = \frac{\Delta p}{\Delta t} = 2m_e c_x \frac{c_x}{2L} = \frac{m_e c_x^2}{L}$
  - Pressure on the wall would be $\frac{F}{A} = \frac{m_e c_x^2}{L} \cdot \frac{1}{L^2} = \frac{m_e c_x^2}{V}$
  - Summing up all the molecules, we get $\frac{1}{3}\frac{m}{V}c_{rms}^2$, where c_{rms} is the root-mean-square velocity
    * Factor of 3 comes from there being 3 components in 3 principle directions
- Now $PV = \frac{1}{3}mc_{rms}^2$
  - As the velocity of molecules increases, momentum and frequency of impact increases, leading to an increase in pressure
- Note $m = NM$ where M is the molar mass, so $PV = \frac{1}{3}NMc_{rms}^2 = NR_uT$
  - This allows us to relate the temperature to what's happening on the molecular level

  - $\frac{1}{2}Mc_{rms}^2 = \frac{3}{2}R_uT$
  - $\frac{1}{2}\frac{M}{N_A}c_{rms}^2 = \frac{1}{2}m_ec_{rms}^2 = \frac{3}{2}\frac{R_u}{N_A}T$
- $k = \frac{R_u}{N_A}$ is the Boltzmann constant

Important

The average kinetic energy of a molecule in the gas is related to the temperature by $\frac{1}{2}m_ec_{rms}^2 = \frac{3}{2}kT$

- This means $\frac{3}{2}nkT$ where n is the number of molecules of gas is the total kinetic energy of the gas

Important

$U = \frac{3}{2}NR_uT$ is the total kinetic energy of all the molecules, or the internal energy

- In mass units this is $\Delta U = \frac{3}{2}mR(T_2 - T_1)$ where R is the gas constant for the specific gas
- This means that a change in temperature is proportional to a change in internal energy
- Compressing a gas does work on it, increasing U, which leads to an increase in T

Important

For all ideal gases, U is a function of T exclusively, and not on pressure

- For any ideal gas $\Delta U = mc(T_2 - T_1)$ where c is the specific heat
  - $c = \frac{1}{m}\frac{\Delta U}{\Delta T}$
- For monoatomic gases we can expect $c = \frac{3}{2}R$