Lecture 25, Nov 14, 2022

Insulation

- The R-value of insulation indicates thermal resistance
  - Note this R value normalized is per unit area (temperature difference per heat flux)
  - $R = \dfrac{\Delta T}{\dot{q}} = \dfrac{L}{k}$, note area is not in here
  - Typically in imperial units in BTU per (hour foot degree Fahrenheit)
- Insulation is typically made of materials containing small air pockets, to reduce conduction and also convection

Critical Radius of Insulation

- Consider a pipe with insulation starting at r_1 and ending at r_2
- The insulation increases the wall thickness; typically increases conductive resistance $R_{cond} = \dfrac{L}{kA}$
- However convective heat resistance is *decreased* due to the increase in area, so insulation can increase heat transfer!
- $\dot{Q} = \dfrac{T_{\infty,1} - T_{\infty,2}}{\frac{\ln \frac{r_1}{r_2}}{2\pi Lk} + \frac{1}{2\pi r_2 Lh}}$
  - There is a critical radius of insulation where $\dot{Q}$ is the maximum
  - The critical point is $r_{crit} = \dfrac{k}{h}$; below this point, insulation increases the heat transfer; above this point insulation decreases it

Heat Flow Through Finned Surfaces

- Assumption: 1D conduction (i.e. temperature is uniform in the y and z directions)
- Fin with cross-sectional area A_c, length L, transferring heat into fluid with T_∞
- Consider an infinitesimal slice of the fin; $\dot{Q}_x$ would not be constant due to heat loss through the sides of the fin
  - Energy balance: $\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv}$
  - Let P be the perimeter of the fin, so the surface area is $P\Delta x$, so $\dot{Q}_{conv} = hP\Delta x(T(x) - T_\infty)$
  - $\dot{Q}_{cond,x+\Delta x} - \dot{Q}_{cond,x} + hP\Delta x(T(x) - T_\infty) = 0$
  - Take $\lim\limits_{\Delta x \to 0}$ and we get $\dfrac{\mathrm{d}Q_{cond}}{\mathrm{d}x} + hP(T(x) - T_\infty) = 0$
  - We know $\dot{Q}_{cond} = -kA_c\dfrac{\mathrm{d}T}{\mathrm{d}x}$ so $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(-kA_c\dfrac{\mathrm{d}T}{\mathrm{d}x}\right) + hP(T(x) - T_\infty) = 0$
  - We usually assume A_c, P, k are constant, so $\dfrac{\partial^2 T}{\partial x^2} - \dfrac{hP}{kA_c}(T(x) - T_\infty) = 0$
- Let $\theta = T - T_\infty$ and $a^2 = \dfrac{hP}{kA_c}$
  - Note $\dfrac{\mathrm{d}\theta}{\mathrm{d}x} = \dfrac{\mathrm{d}T}{\mathrm{d}x}$
  - $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - a^2\theta = 0$
  - $\theta(x) = c_1 e^{ax} + c_2 e^{-ax}$
- Using boundary conditions: $T(0) = T_b$
  - If we assume a fin that's infinitely long, $T(L) = T_\infty \implies \theta(0) = \theta_b, \theta(L) = 0$
    * $\theta(L) = c_1 e^{aL} = 0 \implies c_1 = 0$
    * $\theta(0) = c_2 = \theta_b$
    * $\theta(x) = \theta_b e^{-ax}$ or $\dfrac{\theta(x)}{\theta_b} = e^{-ax}$ or $\dfrac{T(x) - T_\infty}{T_b - T_\infty} = \exp\left(-x\sqrt{\dfrac{hP}{kA_c}}\right)$

* $\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{x=0} = \left.\frac{\mathrm{d}Q}{\mathrm{d}x}\right|_{x=0} = -\theta_b a$
* For an infinitely long fin $\dot{Q} = \sqrt{hPkA_c}(T_b - T_\infty)$

Summary

For an infinitely long fin, the temperature profile varies as:

$$\frac{\theta(x)}{\theta_b} = \frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax}$$

where

$$a = \sqrt{\frac{hP}{kA_c}}$$

where P is the perimeter of the fin, A_c is the cross-sectional area of the fin, and the total heat transfer *through the fin* is

$$\dot{Q} = \sqrt{hPkA_c}(T_b - T_\infty)$$