Lecture 23, Nov 1, 2022

Thermal Resistances in Parallel

- Need to assume each side has the same uniform temperature across all different materials, and heat transfer is only 1D (no heat transfer between the different thermal resistances)
- The total heat flux is $\dot{Q} = \sum_i \dot{Q}_i = \sum_i \frac{T_1 - T_2}{R_i} = (T_1 - T_2)\left(\sum_i \frac{1}{R_i}\right)$
- This gives us $\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$, completely analogous to electrical resistors

Thermal Resistance Networks

- We can combine complex heat transfer conditions into resistance networks
- Simplify using series and parallel resistance rules like in circuits
- Main assumptions:
  - 1D heat flow
  - Isothermal normal to heat flow

Thermal Contact Resistance

- So far we've assumed that at the boundary the temperatures are identical, but this assumes materials are completely flush against each other
- Real surfaces are rough (nano scale topology)
  - Roughness is measured in nanometers
- Due to the roughness the two surfaces are not in perfect contact, so at the boundary there is a slight temperature difference
  - Instead of having $T_1 \to T_2 \to T_3$ we actually have $T_1 \to T_2 \to T_2' \to T_3$
  - Define $\Delta T = T_2 - T_2'$
- Since the air between the layer is a poor conductor in reality most of the heat flow goes through the parts of the surface that are actually in contact
  - The effective heat transfer area is only the area in contact
  - To minimize thermal contact resistances, we can fill in the gaps with a conductive material, e.g. silicon oil, glycerol
    - This is how thermal paste works
- Define the *thermal contact resistance* $R_c = \frac{\Delta T}{\dot{q}}$ with units of $\mathrm{m^2K/W}$
  - Notice this is defined per unit of heat flux, not per unit of heat transfer
  - Same unit as $\frac{1}{h}$ but not $\frac{1}{hA}$ like the other resistances
- Define $h_c = \frac{1}{R_c} = \frac{\dot{q}}{\Delta T}$ as the *thermal contact conductance*
  - $\dot{q} = h_c \Delta T, \dot{Q} = h_c A \Delta T$