Lecture 22, Oct 31, 2022

Thermal Resistance

- Recall in steady-state 1D heat flow we derived $\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = 0 \implies T(x) = \frac{T_2 - T_1}{L}x + T_1$
  - $\dot{q} = -k\frac{\mathrm{d}T}{\mathrm{d}x} = -\frac{k}{L}(T_2 - T_1)$
  - Other boundary conditions:
    1. Known $\dot{q}_1, T_2$
    2. Known T_1, convection boundary condition
- Knowing the temperature profile, we can determine the performance (e.g. heat flux/flow) and other temperature-related properties
- Consider a chip on a circuit board, with a measured power consumption (known $\dot{Q}$ and $\dot{q}$); we also know the thickness of the circuit board, so we have the heat transfer through the circuit board
  - In a real system this is often much more complicated, e.g. the circuit board can have multiple layers and vias, different materials, and a copper heat sink
- The *thermal resistance* approach is a convenient way to analyze complex systems
  - Directly analogous to electrical circuits
  - In a circuit we have flow = driving force divided by resistance
  - In heat transfer $\dot{Q} = \frac{T_1 - T_2}{R}$

Definition

The thermal resistance R is defined such that $\dot{Q} = \frac{T_1 - T_2}{R}$

- For conduction $\dot{Q} = \frac{kA}{L}(T_1 - T_2) \implies R = \frac{L}{kA}$, with units of K/W
- For convection $\dot{Q} = hA(T_1 - T_2) \implies R = \frac{1}{hA}$
- For radiation we have to take a shortcut: $\dot{Q} = \varepsilon\sigma A(T_s^4 - T_{surr}^4)$ is nonlinear, so we force it into $h_{rad}A(T_s - T_{surr})$
  - $h_{rad} = \frac{\varepsilon\sigma A(T_s^4 - T_{surr}^4)}{A(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr})$
  - This a directly a function of T_s and T_{surr}; since T_s is often unknown, we often take a guess and do a question, and then come back later to refine our guess if necessary

Summary

Thermal resistances for the different heat transfer mechanisms in steady state:

- Conduction:
$$R = \frac{L}{kA}$$
- Convection:
$$R = \frac{1}{hA}$$
- Radiation:
$$R = \frac{1}{h_{rad}A}$$
  where
$$h_{rad} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr})$$

- If we have multiple layers of materials in series, we can consider it just like we would consider series resistances in a circuit

- The equivalent heat resistance of multiple layers in series is just the sum of the heat resistances
- We can combine all the $\frac{L}{k}$, $\frac{1}{h}$ and $\frac{1}{h_{rad}}$ into $\frac{1}{U}$ where U is the overall heat transfer coefficient, so $UA = \frac{1}{R_{tot}}$