Lecture 20, Oct 25, 2022

Radiation

- Energy emitted by all matter in the form of electromagnetic waves
  - Thermal radiation is emitted by all bodies above absolute zero
- Typically volumetric, i.e. scales with volume of the body
  - However for opaque objects, radiation can only be emitted from the surface, so radiation scales with surface area
- Amount of radiation is a function of the surface temperature

Equation

Stefan-Boltzmann Law: The maximum amount of radiation that a surface can emit is

$$\dot{Q}_{max} = \sigma A T_s^4$$

where A is the surface area, T_s is the surface temperature, and σ is the Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2\,K^4}$

- For a blackbody, the higher the temperature, the more the distribution of wavelengths shifts towards shorter wavelengths
  - A blackbody is a surface that emits the maximum amount of radiation
  - However real surfaces emit less radiation
- For a real surface, $\dot{Q} = \varepsilon\sigma A T^4$, where ε is the emissivity, $0 \le \varepsilon \le 1$
  - When $\varepsilon = 1$, the body is a blackbody
  - Otherwise it is a *graybody*
- Real objects have complex wavelength distributions that can be approximated by a graybody
  - A graybody has a constant emissivity less than 1
- e.g. the emissivity of black paint is 0.99; aluminum foil has an emissivity of 0.07
- Example: Liquid N_2 is kept inside a vacuum Dewar Flask
  - There are 2 layers of glass separated by a vacuum to prevent conduction and convection
  - The surfaces are coated with silver, which has a very low emissivity, to prevent radiation
- Radiation can also be absorbed when it's incident on a surface
  - Some radiation is absorbed and some is reflected for an opaque system
  - For a blackbody everything is absorbed
  - The fraction absorbed is defined as the *absorptivity* α, such that $\dot{Q}_{\text{absorbed}} = \alpha\dot{Q}_{\text{incident}}$
  - From conservation of energy, the amount reflected is $\dot{Q}_{\text{reflected}} = (1-\alpha)\dot{Q}_{\text{incident}}$
  - Kirchhoff's Law: $\alpha = \varepsilon$
- Special case: when a small surface is completely surrounded by a much larger surface, $\dot{Q}_{net} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4)$
  - This net radiation accounts for radiation absorbed and emitted
- Example: Chip with $\varepsilon = 0.6$ in a room with air/wall temperature 25°C, and $A_s = (0.015\mathrm{m})^2$
  - Natural (free) convection
    - Estimate using a simple model $h = c(T_s - T_\infty)^{\frac{1}{4}}$ where $c = 4.2\ \mathrm{W/m^2\,K^{5/4}}$
  - Forced convection with $h = 250\mathrm{W/m^2\,K}$
  - What is the maximum power we can dissipate if the chip temperature must be less than 85°C?
    - Natural convection: $\dot{Q}_{conv} = hA(T_s - T_\infty) = cA(T_s - T_\infty)^{\frac{1}{4}}(T_s - T_\infty) = 0.158\mathrm{W}$
    - Radiation: $\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4) = 0.065\mathrm{W}$
    - Net heat transfer: $\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = 0.223\mathrm{W}$
      - Note here the convective and radiative heat transfers are of similar magnitude
        - Typically values for free convection is 3-20 $\mathrm{W/m^2\,K}$
    - Forced convection: $\dot{Q}_{conv} = 3.375\mathrm{W}$
      - In this case the radiative heat transfer is only about 2% of the total heat transfer, so we can ignore it

Summary

For heat radiation:

- For an ideal blackbody:

$$\dot{Q} = \sigma A T^4$$

- For a real object:

$$\dot{Q} = \varepsilon \sigma A T^4$$

- If a smaller surface is surrounded by a larger surface:

$$\dot{Q}_{net} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)$$

where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2K^4}$ and ε is the surface emissivity, which is equal to α, the surface absorptivity

Thermal Conductivity Via Electrons

- Wiedemann-Franz Law: relates thermal and electrical conductivities of metals: $\frac{k}{\sigma} = LT$ where σ is the electrical conductivity, k is the thermal conductivity, L is the Lorenz number $L = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 = 2.44 \times 10^{-8}\,\mathrm{V^2/K^2}$

Heat Conduction

- Consider system volume, we can have 3D heat conduction $\dot{Q}_x, \dot{Q}_y, \dot{Q}_z$
  - Temperature is a function of position and time
  - Heat conduction is a vector, $\vec{\dot{Q}} = \dot{Q}_x \hat{\imath} + \dot{Q}_y \hat{\jmath} + \dot{Q}_z \hat{k}$
  - Use Fourier's law: $\dot{Q}_x = -kA_x \frac{\mathrm{d}T}{\mathrm{d}x}, \dot{Q}_y = -kA_y \frac{\mathrm{d}T}{\mathrm{d}y}, \dot{Q}_z = -kA_z \frac{\mathrm{d}T}{\mathrm{d}z}$
- We will assume our system is 1 dimensional
  - This can happen if $\Delta x \ll \Delta y, \Delta z$ (e.g. a wall or through a plate), so $\frac{\mathrm{d}T}{\mathrm{d}x} \gg \frac{\mathrm{d}T}{\mathrm{d}y}, \frac{\mathrm{d}T}{\mathrm{d}z}$ and we can ignore the other two dimensions
  - This can also happen if we have insulated sides
  - Can also happen in cylindrical coordinates