Lecture 19, Oct 24, 2022

Conduction

- Heat transfer through a stationary medium as a result of a temperature difference
- Temperature T_1 on one side and T_2 on the other, resulting in a heat transfer $\dot{Q}$
- Conduction can occur in any material – solid, liquid or gas, but without long range motion in the medium
- "Thermometers are speedometers for atoms"
  - In a solid, atoms can vibrate in their lattice
  - In a fluid, atoms and molecules can translate, molecules can vibrate and rotate
  - At higher temperature, atoms move faster; collisions between them transfer energy, which is heat transfer
- Conduction is modelled by *Fourier's Law*
  - Consider a temperature gradient T_1, T_2; somewhere in the middle we have T_0
  - Heat flux $\dot{q} = \dfrac{\dot{Q}}{A}$, with $\dot{q}^+$ from T_0 to T_2 and $\dot{q}^-$ from T_0 to T_1
  - Define the average molecular velocity $\bar{v}$, number density n, and mean free path λ
    - Over a distance of λ the direction of the molecules should be constant
  - Molecular energy is $mcT(y)$ where m is the molecular mass, c is the specific heat capacity and $T(y)$ is temperature
  - The heat flux is defined as the number of molecules crossing a unit area per unit time, times the number energy per molecule
    - $\dot{q} = n\bar{v}mcT(y)$
    - The positive heat flux is $\dot{q}^+ = n\bar{v}mcT\left(-\dfrac{\lambda}{2}\right)$
    - The negative flux is $\dot{q}^- = n\bar{v}mcT\left(\dfrac{\lambda}{2}\right)$
    - The net heat flux is the difference, $n\bar{v}mc\left(T\left(-\dfrac{\lambda}{2}\right) - T\left(\dfrac{\lambda}{2}\right)\right)$
    - We can approximate $T\left(-\dfrac{\lambda}{2}\right) - T\left(\dfrac{\lambda}{2}\right)$ as $-\lambda\dfrac{\mathrm{d}T}{\mathrm{d}y}$
  - $\dot{q} = -n\bar{v}mc\lambda\dfrac{\mathrm{d}T}{\mathrm{d}y}$
  - The first 5 constants are all properties of the gas, which we define to be k, the thermal conductivity

Equation

Fourier's Law:

$$\dot{Q} = \dot{q}A = -kA\frac{\mathrm{d}T}{\mathrm{d}y}$$

where the thermal conductivity $k = n\bar{v}mc\lambda$, where n is the number density, $\bar{v}$ is the velocity, m is the mass per molecule, c is the heat capacity and λ is the mean free path

In reality $k = k(T, P)$, but we assume it to be constant

- Notes:
  - k has units of W$m.K$
  - The sign is negative – the direction of heat conduction is opposite to the temperature gradient
  - The closer the atoms are, the better the thermal conductivity
    - Typically k is the greatest for solids, then liquids, then gases
- In addition to transfer of kinetic energy, conduction can also occur through electron flow
  - This is why good electrical conductors are usually good thermal conductors

Convection

- Heat transfer between a solid surface and a moving fluid
- Two heat transfer mechanisms:
  1. Motion/collision of fluid molecules (conduction)
  2. Energy transfer due to bulk motion of fluid (advection)
- We care about fluid flow near surfaces
  - Near the surface of the solid, we have the *boundary layer*, where the fluid is severely slowed down
  - At the surface we have a no-slip condition, i.e. the fluid has zero velocity
  - The edge of the boundary later is defined as where $v = 0.99v_{free}$ where v_{free} is the free stream velocity
- When Re < 2000, the flow is laminar
- Temperature will also have a boundary layer – the temperature as you approach the surface differers from that of the bulk
  - $\dot{Q}_{conv} = -kA\frac{\mathrm{d}T}{\mathrm{d}y}$, but $\frac{\mathrm{d}T}{\mathrm{d}y}$ is a complex function of fluid mechanics
  - We typically use Newton's Law of Cooling, $\dot{Q}_{conv} = hA(T_s - T_\infty)$, where h is the heat transfer coefficient in units of $\mathrm{W/m^2\,K}$
    * h is a function of fluid flow and properties
      - Thermal conductivity of the fluid
      - Surface geometry
      - Fluid velocity (higher average velocity leads to a higher h)

Equation

Newton's Law of Cooling: $\dot{Q}_{conv} = hA(T_s - T_\infty)$