

# Lecture 16, Oct 17, 2022

## Heat Engines

- What is the most efficient engine possible?
- A heat engine is any device that works in a thermodynamic cycle which does work on its surroundings as long as heat is supplied
  - The engine must do a net amount of work (that is, it can use work internally, but it must produce work output)
  - It must be able to work continuously (so energy storage devices do not count)
- We can model a heat as a system that takes heat  $Q_H$  from a hot thermal reservoir  $T_H$ , puts heat  $Q_C$  into a cold thermal reservoir  $T_C$  while producing work  $W_{net}$
- Define the thermal efficiency as  $\eta_{th} = \frac{W_{net}}{Q_H}$  (neglecting sign convention)
  - So what is the maximum possible efficiency?
- A note on perpetual motion machines:
  - PMM of the first kind violate the first law (they create energy from nothing)
  - PMM of the second kind violate the second law
    - \* This is a little more subtle to see
    - \* e.g. an engine that directly converts heat to work, without the use of a cold thermal reservoir
      - Over a thermodynamic cycle, all properties go to their initial values
      - However in this engine, heat only comes in, so the entropy can only increase and is never removed from the system, making it an invalid heat engine
      - This is why heat engines must have a cold reservoir to reject heat in order to remove entropy
- Kelvin-Planck statement: It is impossible for any device operating in a thermodynamic cycle to receive heat from a high temperature source and produce device work without rejecting heat to a low temperature sink
  - This can be an alternative statement of the second law

## Carnot Engine

- A Carnot engine is the theoretically most efficient heat engine possible
- Consider the same system that takes heat  $Q_H$  from a hot thermal reservoir  $T_H$ , puts heat  $Q_C$  into a cold thermal reservoir  $T_C$  while producing work  $W_{net}$
- All processes are reversible; no  $s_{gen}$ , the engine is frictionless, and all heat and work transfer is reversible
- Over a cycle  $\Delta E = 0 \implies Q_H - Q_C - W = 0 \implies W = Q_H - Q_C$
- $\Delta S = 0 \implies \oint \frac{\delta Q_{rev}}{T} = 0$ ; this can be broken down into the entropy from heat addition and heat rejection
- We want to minimize the entropy added during heat addition, because we need to get rid of it later
  - This means we need to maximize the temperature, so add heat when  $T = T_H$ , which means the heat transfer is isothermal
  - $\Delta S_{\text{heat addition}} = \frac{Q_H}{T_H}$
- We also want to maximize the entropy removed during heat rejection
  - This means we need to reject heat at the lowest temperature, so reject heat at  $T = T_C$
  - $\Delta S_{\text{heat rejection}} = \frac{Q_C}{T_C}$
- To close the thermodynamic cycle, temperature change is done first through an isentropic expansion, then an isentropic compression
- $\Delta S_{\text{heat addition}} - \Delta S_{\text{heat rejection}} = 0 \implies \frac{Q_H}{T_H} = \frac{Q_C}{T_C}$  and  $W_{net} = Q_H - Q_C$ 
  - $\eta_{th} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$

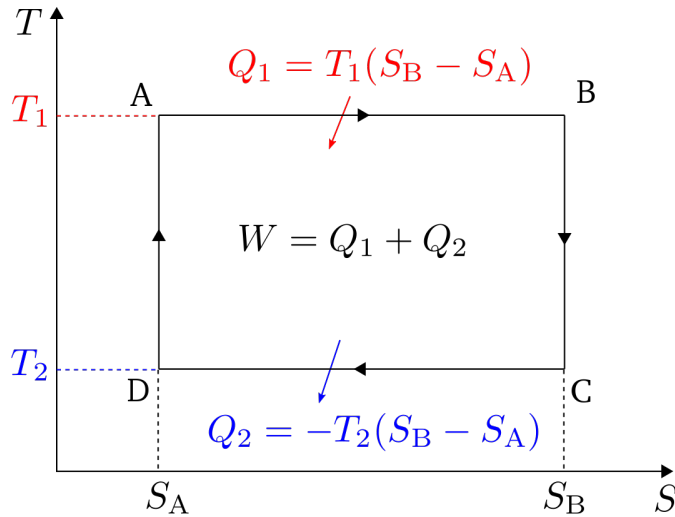


Figure 1: Carnot engine on a  $T$ - $S$  diagram

### Definition

The Carnot Efficiency:  $\eta_{th} = 1 - \frac{T_C}{T_H}$  is the theoretical maximum efficiency of a heat engine

- This means the maximum efficiency of an engine depends only on the temperatures of the thermal reservoirs and not the engine cycle
  - $T_C$  is usually fixed, so the higher we raise  $T_H$ , the better our efficiency

### Implementing a Carnot Cycle

- Practically we can do a two-phase Carnot cycle
- Isothermal heat addition can be accomplished with a boiler
  - If we find a liquid that boils at  $T_H - \Delta T$ , we can pipe the liquid into the boiler, which takes heat from the heat source and the liquid emerges as vapour
- The resulting vapour can be passed into an isentropic turbine, which cools the liquid, extracting work and producing a liquid-vapour mixture
- The heat rejection can be accomplished with a condenser
  - The liquid-vapour mixture would be at temperature  $T_C + \Delta T$ , which rejects heat to a cold heat sink and produces a liquid
- Finally the liquid is is passed into an isentropic compressor (which requires work, but can be driven by the turbine) and the cycle is complete

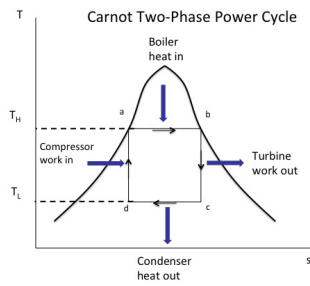


Figure 2: Two-phase Carnot cycle on a  $T$ - $S$  diagram