Lecture 12, Oct 4, 2022

More on Entropy Balance

- Recall $\frac{\mathrm{d}S}{\mathrm{d}t} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen}$
- Consider a control volume where $\dot{m}_i, \dot{m}_e$ are the mass entry and exit rates
  - In addition to the heat transfer, the internal irreversibilities generating entropy, there is also entropy carried in by the mass
  - Entropy balance is then $\frac{\mathrm{d}s}{\mathrm{d}t} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{s}_{gen}$
    * Reversible system means $\dot{s}_{gen} = 0$
    * Adiabatic process has $\dot{Q} = 0$
    * Steady state systems have $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$
    * With single-entry and single-exit systems $\dot{m}_i = \dot{m}_e$

Summary

Entropy balance for a control mass: $\frac{\mathrm{d}s}{\mathrm{d}t} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{s}_{gen}$

For a control volume: $\frac{\mathrm{d}s}{\mathrm{d}t} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{s}_{gen}$

Isentropic Steady Flow Devices

- Consider a burner outputting $\dot{m}, P_1, T_1$, how much work can we get out of it if we connect this to a turbine?
  - We know the inlet conditions, but we can't simply do $\dot{m}c_p(T_2 - T_1)$ because we don't have T_2
  - We know $P_2 = P_{atm}$
  - In the most ideal case, if we consider the turbine to have no internal irreversibilities and completely adiabatic, then the process is isentropic and $s_1 = s_2$
  - This allows us to get the work in the most ideal case

Isentropic Efficiency

- In real life nothing is really isentropic, so how do we relate the numbers for the most ideal case to real life?
- Recall $s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$
  - Assume constant pressure then $T_2 = T_1 \exp\left(\frac{s_2 - s_1}{c_p}\right)$
- The curve of T with respect to s depends on P; at P_1 and P_2 there are two separate curves
  - By dropping down from T_1, s_1 vertically (isentropic) we can find T_2
  - In reality (non-isentropic process) we'd have to move to the right since entropy increases, which gives us a different (higher) T_2

Definition

Isentropic efficiency for a turbine: $\eta_t = \dfrac{\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{actual}}}{\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{ideal}}}$

For a compressor this would be $\eta_c = \dfrac{\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{ideal}}}{\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{actual}}}$

- Assume constant c_p then $\eta_t = \dfrac{h_2 - h_1}{h_{2s} - h_1} = \dfrac{c_p(T_2 - T_1)}{c_p(T_{2s} - T_1)} = \dfrac{T_2 - T_1}{T_{2s} - T_1}$
  - This is easy to measure and is typically provided by a turbine manufacturer (typically 80% to 90%)
- Do this in reverse and it works for a compressor
- For a nozzle it's defined in terms of KE: $\eta_{nozzle} = \dfrac{\frac{V_2^2}{2}}{\frac{V_{2s}^2}{2}}$

Bernoulli's Equation

- Assume *inviscid* flow (no viscosity, no friction), so it can be modelled as a reversible flow
- Consider fluid entering a pipe at V_1, P_1, z_1, T_1 and exiting at V_2, P_2, z_2, T_2
- Assume flow is isentropic, incompressible
  - This is generally true for gases that aren't moving too fast
- This means $\Delta s = c \ln \dfrac{T_2}{T_1} = 0 \implies T_2 = T_1$
- $h_2 - h_1 = c(T_2 - T_1) + v(P_2 - P_1) = v(P_2 - P_1)$
- The energy balance equation becomes $\dfrac{P_2 - P_1}{\rho} + \dfrac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$
  - Rearrange to $\dfrac{P_1}{\rho} + \dfrac{V_1^2}{2} + gz_1 = \dfrac{P_2}{\rho} + \dfrac{V_2^2}{2} + gz_2$
  - This means this is a constant

Definition

Bernoulli's Equation: $\dfrac{P}{\rho} + \dfrac{V^2}{2} + gz = \text{const}$ (where V is the velocity)

- If we assume z is roughly constant, then as velocity goes up, pressure goes down