Lecture 11, Oct 3, 2022

Isentropic Processes

- For incompressible substances $\Delta s = c_{avg} \ln \frac{T_2}{T_1}$, then for an isentropic process $\Delta s = 0 \implies T_2 = T_1$
- Internal irreversibility: something within the system converting work into heat
  - An isentropic system is internally reversible
  - In some process, the less entropy you generate, the better it is (the ideal case would be completely isentropic)
- For an ideal gas $\Delta s = 0 \implies c_v \ln \frac{T_2}{T_1} = -R \ln \frac{v_2}{v_1} \implies \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\frac{R}{c_v}}$
  - Recall $\gamma = \frac{c_p}{c_v}$ and $c_p - c_v = R$ so $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$
  - We can also show that $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$
  - Combining the two we get $\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \implies P_1 v_1^{\gamma} = P_2 v_2^{\gamma} = \text{const}$
  - This is the equation for a polytropic process!

Important

A polytropic process ($Pv^n = \text{const}$) is isentropic if $n = \gamma$

Summary

For an isentropic process:

- $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$
- $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$
- $\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma}$

Where $\gamma = \frac{c_p}{c_v}$

Entropy Balance in a Control Mass

- $\Delta S = S_{in} - S_{out} + S_{gen}$, at equilibrium this is equal to 0
  - Entropy can be transferred in by heating the system and transferred out by cooling it down
- If the system is internally reversible, then $\Delta S = S_{in} - S_{out}$
- On a rate basis $\frac{\mathrm{d}S}{\mathrm{d}t} = \dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen}$
  - For a control mass the only way entropy can be transferred is via heat, so $\dot{S}_{heat} = \frac{\dot{Q}}{T}$ (T is the temperature of the boundary where heat crosses)
  - $\frac{\mathrm{d}S}{\mathrm{d}t} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen}$ where T_j are the local temperatures of the boundaries at which the heat crosses