Tutorial 1, Nov 3, 2022

The Need For Dimensional Analysis

- Can help us do fewer experiments by grouping them
- Allows us to extrapolate experiments on a small scale to results on a much larger scale

Dimensions

- *Dimension*: A qualitative description of a physical quantity
- $[\cdots]$ means to take the dimensions of
- There are two types of dimensions:
  - Primary dimensions: dimension that are not formed by other dimensions
    - e.g. mass, length, time (MLT): $[\text{mass}] = M, [\text{length}] = L, [\text{time}] = T$
    - We can also choose other systems of primary dimensions, e.g. force, length, time
      - However we can't include force in the MLT system since force can be expressed as a combination of mass, length and time
    - In heat transfer we also have temperature $[\text{temperature}] = \theta$
  - Secondary dimensions: dimensions that are derived from the primary dimensions
    - e.g. density or pressure in the MLT system
      - $[\rho] = \dfrac{M}{L^3}$
      - In the FLT system $[\rho] = \dfrac{FT^2}{L}\dfrac{1}{L^3} = \dfrac{FT^2}{L^4}$
- Buckingham Pi Theorem: The number of π terms is equal to the number of variables minus the minimum number of required reference dimensions
  - We call dimensionless numbers π terms, e.g. $\pi_1 = \dfrac{L}{D}, \pi_2 = \dfrac{AD}{C^2}$
  - Reference dimensions are *usually* the primary dimensions
  - e.g. if $A = f_1(B, C, D, E, F)$ and we describe this system using the reference dimensions M, L, T, then the number of π terms is $6 - 3 = 3$
    - This allows us to say $\pi_1 = f_2(\pi_2, \pi_3)$
    - This allowed us to simplify the problem so it's easier for us to do experiments
- To determine π terms we use the method of repeating variables

Method of Repeating Variables

1. List all the *independent* variables that are involved in the problem
   - e.g. velocity, density, length
   - They have to be independent, e.g. we can't have both diameter and cross-sectional area of a pipe
2. Express each variable in terms of primary dimensions
   - e.g. express density as $[\rho] = ML^{-3}$ in MLT
3. Determine the required number of Pi terms using the Buckingham Pi Theorem
   - e.g. $[A] = \dfrac{M}{L^3}, [B] = \dfrac{M}{L^3T^2}, [C] = \dfrac{MT}{L^3}$
4. Choose some repeating variables, such that the number of repeating variables is equal to the number of reference dimensions
   - The choice of repeating variables is fairly arbitrary, except:
     - We can't select the dependent variable
     - The repeating variables must cover all reference dimensions
     - Each variable must be dimensionally independent (i.e. you can't construct one dimension by multiplying together or dividing others)
   - e.g. if we have $A = f(B, C, D, E, F)$ in MLT, then we need to select 3 repeating variables, but we can't pick A and the ones we pick must cover M, L, and T
5. Form a π term by multiplying each of the non-repeating variables by some combination of the repeating variables raised to powers such that the combination is dimensionless

- e.g. if we select B, C, D as repeating variables, we need to do so for A, E, F
  - $\pi_i = AB^{x_i}C^{y_i}D^{z_i}$
  - We need to select x_i, y_i, z_i such that the combination is dimensionless

6. Check that the π terms are dimensionless
7. Express a new relationship among π terms
   - e.g. $A = f_1(B, C, D, E, F) \implies \pi_1 = f_2(\pi_2, \pi_3)$

Example

- We would like to find the pressure difference per unit length with a flow with velocity V, density ρ, viscosity μ, between two pressures p_1 and p_2 separated by distance L
- Using the method of repeating variables:
  1. Variables:
     - $\dfrac{\Delta p}{L} = f(V, D, \rho, \mu)$
  2. Expressing in primary dimensions:
     - $\left[\dfrac{\Delta p}{L}\right] = \dfrac{ML}{T^2}\dfrac{1}{L^2}\dfrac{1}{L} = \dfrac{M}{T^2L^2}$
     - $[V] = \dfrac{L}{T}$
     - $[D] = L$
     - $[\rho] = \dfrac{M}{L^3}$
     - $[\mu] = \dfrac{M}{LT}$
  3. Buckingham Pi Theorem
     - 5 variables, 3 reference dimensions gives us 2 Pi terms
  4. Choose repeating variables
     - We need to choose 3
     - We can choose ρ, V, D
  5. Form Pi terms
     - We need 2 π terms
     - $\pi_1 = \dfrac{\Delta p}{L}\rho^a V^b D^c = \dfrac{M}{T^2L^2}\dfrac{M^a}{L^{3a}}\dfrac{L^b}{T^b}L^c = M^{1+a}L^{-2-3a+b+c}T^{-2-b}$
       * This gives a system $\begin{cases} 1 + a = 0 \\ -2 - 3a + b + c = 0 \\ -2 - b = 0 \end{cases}$ which gives $a = -1, b = -2, c = 1$
       * $\pi_1 = \dfrac{\left(\frac{\Delta p}{L}\right) D}{\rho V^2}$
     - For π_2 we can use a similar process
       * $\pi_2 = \dfrac{\mu}{\rho V D}$
  6. Check dimensions of π terms
  7. Express the new relationship
     - $\pi_1 = f_2(\pi_2)$
     - $\dfrac{\left(\frac{\Delta p}{L}\right) D}{\rho V^2} = f_2\left(\dfrac{\mu}{\rho V D}\right)$
     - Now we only need to find one relationship instead of the 3 relationships before!