# Lecture 9, Sep 23, 2022

## Cylindrical Coordinates

- Uses triplets of $(r, \theta, z)$
  - $z$ is the distance from the $r\theta$ plane, same as Cartesian $z$
  - $r, \theta$ work like polar coordinates
- Conversion: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$

**Important**

If $f(x, y, z)$ is continuous in

$$Q = \{\, (x, y, z) \mid (x, y) \in R, u_1(x, y) \leq z \leq u_2(x, y) \,\}$$

where

$$R = \{\, (r, \theta) \mid \alpha \leq \theta \leq \beta, h_1(\theta) \leq r \leq h_2(\theta) \,\}$$

then

$$\begin{aligned} \iiint_Q f(x, y, z)\, \mathrm{d}V &= \iint_R \int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\, \mathrm{d}z\, \mathrm{d}A \\ &= \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\, \mathrm{d}z\, \mathrm{d}r\, \mathrm{d}\theta \end{aligned}$$

- Note in cylindrical coordinates, $\mathrm{d}V = r\, \mathrm{d}z\, \mathrm{d}r\, \mathrm{d}\theta$

## Spherical Coordinates

- Uses triplets of $(\rho, \theta, \phi)$
  - $\rho$ is the distance from the origin, always non-negative
  - $\phi$ is the angle from the $z$ axis
    - $\phi$ is between 0 (straight up) and $\pi$ (straight down)
  - $\theta$ is the angle from the $x$ axis, in the $xy$ plane
- Conversion: $\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$
  - $\rho\sin\theta$ is $r$ in the $xy$ plane
- Constant $\rho$: sphere
- Constant $\theta$: vertical plane
- Constant $\phi$: cone
- Therefore in spherical coordinates $\mathrm{d}V = \rho^2 \sin\phi\, \mathrm{d}\rho\, \mathrm{d}\phi\, \mathrm{d}\theta$

**Important**

Triple integration in spherical coordinates:

$$\iiint_Q f(x, y, z)\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z = \iiint_{Q'} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\, \rho^2 \sin\phi\, \mathrm{d}\rho\, \mathrm{d}\phi\, \mathrm{d}\theta$$