Lecture 8, Sep 23, 2022

Applications of Triple Integrals

- Finding mass: If $\rho = \rho(x, y, z)$ is the mass per unit volume over a region Q, then $m = \iiint_Q \rho(x, y, z)\,\mathrm{d}V$
- The centres of mass:
  - $M_{yz} = \iiint_Q x\rho(x, y, z)\,\mathrm{d}V = m\bar{x} \implies \bar{x} = \dfrac{M_{yz}}{m}$
  - $M_{xz} = \iiint_Q y\rho(x, y, z)\,\mathrm{d}V = m\bar{y} \implies \bar{y} = \dfrac{M_{xz}}{m}$
  - $M_{xy} = \iiint_Q z\rho(x, y, z)\,\mathrm{d}V = m\bar{z} \implies \bar{z} = \dfrac{M_{xy}}{m}$
- The centroids:
  - $x_c = \dfrac{1}{V}\iiint_Q x\,\mathrm{d}V$
  - $y_c = \dfrac{1}{V}\iiint_Q y\,\mathrm{d}V$
  - $z_c = \dfrac{1}{V}\iiint_Q z\,\mathrm{d}V$
- Moment of inertia:
  - $I = \iiint_Q \rho(x, y, z)(r(x, y, z))^2\,\mathrm{d}V$ where r is the distance to the axis of rotation