# Lecture 7, Sep 22, 2022

## Triple Integrals

- Many ideas from double integrals carry over
- Given a continuous $w = f(x, y, z)$ over a 3D region $Q$ with volume $V$, break $V$ into sub-volumes $\Delta V_i$, choose any sample point within the region $P_i(x_i^*, y_i^*, z_i^*) \in \Delta V_i$, then
  - Define the lower sum $\sum_{i=1}^{n} m_i \Delta V_i$ and upper sum $\sum_{i=1}^{n} M_i \Delta V_i$
  - If the function is continuous then $\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x_i^*, y_i^*, z_i^*) \Delta V_i = \iiint_Q f(x, y, z) \, \mathrm{d}V$
  - Note $\|P\|$ is now the largest diameter of all subvolumes
- Sometimes only one integration sign is used
- In Cartesian coordinates, $\iiint_Q f(x, y, z) \, \mathrm{d}V = \iiint_Q f(x, y, z) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$
- Suppose $f(x, y, z)$ is continuous over $Q = \{ (x, y, z) \mid a \leq x \leq b, c \leq y \leq d, r \leq z \leq s \}$, then $\iiint_Q f(x, y, z) \, \mathrm{d}V = \int_r^s \int_c^d \int_a^b f(x, y, z) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$
  - As with the double integral case, the order of integration can be freely rearranged
  - If $Q = \{ (x, y, z) \mid (x, y) \in R, g_1(x, y) \leq z \leq g_2(x, y) \}$, then $\iiint_Q f(x, y, z) \, \mathrm{d}V = \iint_R \int_{g_1(x,y)}^{g_2(x,y)} f(x, y, z) \, \mathrm{d}z \, \mathrm{d}A$