Lecture 6, Sep 16, 2022

Surface Area

- Given $z = f(x, y)$ on region R, how do we find its surface area?
- Divide the surface into many subregions S_i with area ΔS_i
- For each subregion draw a tangent plane, the elementary region has area ΔT_i (think of a disco ball)
- $S \approx \sum_{i=1}^{n} \Delta T_i \implies S = \lim_{\|P\| \to 0} \sum_{i=1}^{n} \Delta T_i \implies S = \iint_S \mathrm{d}T$
- Project subregion T_i with area ΔT_i down to R and get a rectangular subregion R_i with area $\Delta A_i = \Delta y_i \Delta x_i$
  - ΔT_i is a parallelogram
  - The two vectors that define this parallelogram are $\vec{a}_i, \vec{b}_i$
  - $\vec{a}_i$ has slope $f_x(x_i, y_i)$, $\vec{b}_i$ has slope $f_y(x_i, y_i) \implies \vec{a}_i = \begin{bmatrix} \Delta x_i \\ 0 \\ f_x(x_i, y_i)\Delta x_i \end{bmatrix}, \vec{b}_i = \begin{bmatrix} 0 \\ \Delta y_i \\ f_y(x_i, y_i)\Delta y_i \end{bmatrix}$
- Cross product to get area: $\vec{a}_i \times \vec{b}_i = \begin{bmatrix} -f_x(x_i, y_i) \\ -f_y(x_i, y_i) \\ 1 \end{bmatrix} \Delta x_i \Delta y_i$
  - $\Delta T_i = \|\vec{a}_i \times \vec{b}_i\| = \sqrt{(f_x(x, y))^2 + (f_y(x, y))^2 + 1} \Delta x_i \Delta y_i$, and now we can integrate to get surface area

Important

The surface area of $z = f(x, y)$ on R is

$$S = \iint_R \sqrt{(f_x(x, y))^2 + (f_y(x, y))^2 + 1} \, \mathrm{d}A$$