Lecture 5, Sep 16, 2022

Applications of Double Integrals

- Consider a thin plate with density $\rho = \rho(x, y)$ (with dimensions of M/A)
  - We can find the plate's total mass with $m = \iint_R \rho(x, y)\,\mathrm{d}A$
- We can find the plate's centre of mass by $\bar{x} = \frac{1}{A}\iint_R x\rho(x, y)\,\mathrm{d}A, \bar{y} = \frac{1}{A}\iint_R y\rho(x, y)\,\mathrm{d}A$
- For centroids: $x_c = \frac{1}{A}\iint_R x\,\mathrm{d}A, y_c = \frac{1}{A}\iint_R y\,\mathrm{d}A$
  - Like centre of mass, but uniform $\rho = 1$
- Moment of inertia: recall $I = \sum_{i=1}^{n} m_i r_i^2$
  - As an integral this is $I_0 = \iint_R \rho(x, y)(x^2 + y^2)\,\mathrm{d}A$ (note I_0 is the moment of inertia about the origin)
  - Note this means we can separate $I_0 = \iint_R \rho(x, y)x^2\,\mathrm{d}A + \iint_R \rho(x, y)y^2\,\mathrm{d}A = I_x + I_y$