Lecture 36, Dec 2, 2022

Open Channel Flows

- Flows of liquid with free surface exposed to atmospheric pressure pressure
  - This free surface introduces an extra degree of freedom
  - This allows aves to form
- Waves move at a speed of c_0, which is not the same as the velocity of the individual fluid particles
- We want a control volume that moves with the wavefront
  - Assuming a wave height much less than the liquid height, $\delta y \ll y$, then $c_0 = \sqrt{gy}$
  - The wave speed depends on the liquid depth; this is why tsunamis form, since waver level is very deep in the ocean
- For open channel flows, we define the Froude number $\mathrm{Fr} = \dfrac{V}{\sqrt{gy}}$, the ratio of the fluid speed divided by the wave speed; the Froude number governs the character of the flow in open channels
  - $\mathrm{Fr} < 1$ is subcritical flow
    - The waves drift due to the velocity, but the wave still moves both down and upstream since the wave is able to travel faster than the fluid
  - $\mathrm{Fr} = 1$ is critical flow
    - The wave velocity matches the fluid velocity, so the wavefront stays in place
  - $\mathrm{Fr} > 1$ is supercritical flow
    - The waves only move downstream since the fluid pushes it down faster than it can go upstream
- Open channel flows are similar to compressible flows, in which the Mach number is used
  - In supercritical flow the wavefront is analogous to the shockwave in supersonic flow

Compressible Flows

- Incompressibility is always only an approximation
  - The constant density assumption greatly simplifies problems
  - This is valid in a slow moving fluid
- In compressible flows we need fluid dynamics and thermodynamics
- A weak pressure wave is defined as a sound wave
  - The pressure wave is travelling at the speed of sound, but not the fluid particles
- Like in the open channel flow we again look at a control volume moving with the wavefront and assume 1D travel
  - Using continuity and momentum we get $c^2 = \left(\dfrac{\partial p}{\partial \rho}\right)_s$
    - Note this is at constant entropy, because the disturbance is very small and we're not adding heat
- For an ideal gas, $\dfrac{P_2}{P_1} = \left(\dfrac{\rho_2}{\rho_1}\right)^\gamma = \left(\dfrac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$
  - $\dfrac{p}{\rho^\gamma}$ is constant
  - $c = \sqrt{\gamma RT}$ for an ideal gas
- More generally for any fluid we can use the bulk modulus and get $c = \sqrt{\dfrac{E_v}{\rho}}$
- In a truly incompressible medium, $E_v \to \infty$ which means $c \to \infty$
- The Mach number is defined as $M = \dfrac{V}{c}$
  - Note this is a variable from point to point
  - We generally use M_∞, the free stream Mach number
- We can categorize the flow based on Mach number:
  - $M_\infty \leq 0.3$ means the flow is incompressible
  - $M_\infty > 0.3$ means the flow is incompressible
  - $0.8 \leq M_\infty \leq 1$ gives transonic flow

  - $M_\infty \geq 5$ gives hypersonic flow

Simplified Compressible Flows

- We will assume steady, 1D, isentropic (adiabatic and inviscid), and compressible flow
- ρVA is constant, so we have $\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}A}{A} + \frac{\mathrm{d}V}{V} = 0$ as an alternative form of the continuity equation
- Pressure work can be derived as $p_1 A_1 V_1 - p_2 A_2 V_2$
- Using RTT on energy balance, we get the compressible Bernoulli equation: $\frac{p}{\rho} + e + \frac{V^2}{2} + gz = \text{const}$ where e is the total internal energy per unit mass
  - In terms of enthalpy $h + \frac{V^2}{2} + gz = \text{const}$
- For high-speed flows, potential energy of the fluid is negligible; if we imagine that we can adiabatically slow the fluid to zero, then we get $h + \frac{v^2}{2} = h_0$, the *stagnation enthalpy* or *total enthalpy*
  - Kinetic energy converts to enthalpy
  - All the kinetic energy goes to an increase in internal energy (temperature) and pressure energy
- We can find properties of the fluid at stagnation:
  - $c_p(T - T_0) + \frac{V^2}{2} = 0 \implies T_0 = T + \frac{V^2}{2c_p}$
  - T is the static temperature, the regular temperature we know
  - $\frac{V^2}{2c_p}$ is the dynamic temperature, the temperature rise in the stagnation process
  - T_0 is the stagnation or total temperature, the temperature we get when we bring the fluid to a stop adiabatically
  - For a very high speed flow we have $T_0 > T$ and kinetic energy is important, but for low speed flows we have $T_0 \approx T$ since kinetic energy is negligible
- We can get properties such as the stagnation temperature in terms of the mach number (formula in notes)
  - At very high velocities, the stagnation temperature can be significantly higher than the free stream temperature
  - Shockwaves will dissipate some of this temperature
- The rule of thumb is we must take compressibility into account when the density changes are greater than 5%
  - If we use the formula of $\frac{\rho_0}{\rho}$ from M, we get a mach number of about 0.3 as the critical threshold

Variation of Fluid Velocity With Flow Area

- Using the continuity equation and conservation of energy we can derive $\frac{\mathrm{d}A}{A} = -\frac{\mathrm{d}V}{V}(1 - M^2)$
  - In subsonic flow, we get $\frac{\mathrm{d}A}{\mathrm{d}V} < 0$ – velocity decreases with increasing area
    * We know this from the incompressible Bernoulli equation
  - In supersonic flow, $\frac{\mathrm{d}A}{\mathrm{d}V} > 0$ – velocity increases with increasing area
    * Density rapidly decreases so the air fills the entire channel
  - In sonic flow, $\frac{\mathrm{d}A}{\mathrm{d}V} = 0$ which means $\mathrm{d}A = 0$
    * If we have area as a function of x, then we have $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$, which means the point at which we have sonic flow must be either a point of maximum or minimum area
    * Since $\frac{\mathrm{d}A}{\mathrm{d}V} < 0$ for subsonic flow, the only way we can get this is to have a converging-diverging duct; then we get $M = 1$ at the minimum point of the duct (the throat)
    * After the throat, the flow can accelerate to supersonic (this depends on the duct design and the pressure after the duct)