Lecture 31, Nov 24, 2022

Fluid in Motion (Continued)

- Viscous regions are regions in which the frictional effects are significant
  - In these regions we need to consider shear forces due viscous effects
  - Boundary layers are viscous regions; in these areas the friction slows down the velocity and creates a gradient
  - This also includes wake regions
- Inviscid regions are regions in which the frictional effects are negligible
  - Doesn't mean that viscosity is zero, but we don't have to consider shear stresses
- Flow dimensionality: a flow is n-dimensional if flow properties vary in n dimensions
  - e.g. consider a uniform flow entering a pipe; near the entrance region, the flow is 2 dimensional since the velocity profile changes as you go down the pipe; after the flow is fully developed, the flow becomes 1 dimensional as the velocity profile stops changing
  - This depends on what coordinate system is used! e.g. with the pipe example, the flow is 2 or 1 dimensional in cylindrical coordinates, but is 3 or 2 dimensional in Cartesian coordinates
- Laminar flow: highly ordered fluid motion with smooth layers
- Turbulent flow: Highly disordered fluid motion with a lot of velocity fluctuations
  - Typically occurs at high velocities
- Transitional flow: a flow that alternates between laminar and turbulent

Definition

The Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho V L}{\mu}$$

At low Reynolds numbers, flow tends to be laminar; at high Reynolds numbers, flow tends to be turbulent

- The Reynolds number is a non-dimensional number that characterizes the flow; most fluids have a critical Reynolds number at which they transition from laminar to turbulent
  - A ratio of inertial to viscous forces
  - Higher viscosity makes the flow more laminar
  - Higher velocity, density or characteristic length makes the flow turbulent

Mass Conservation

- For now, assume steady (time-independent), incompressible (constant ρ) and 1 dimensional flow
- The volumetric flow rate $\dot{V\!\!\!/}$ is the rate of fluid passing through a given region per unit time, in units of $\mathrm{m^3/s}$
- The volumetric flow rate through a surface A is $V\!\!\!/ = \iint_A \vec{V} \cdot \vec{n}\,\mathrm{d}A = \iint_A V_n\,\mathrm{d}A$ where V_n is the normal component of velocity
  - In the special case where V_n is constant, $\dot{V\!\!\!/} = V_n A$
  - Multiply by ρ for the mass flow rate $\dot{m}$
- Since the flow is steady, the volume flow rate along a streamtube does not change therefore we have $\dot{m}_1 = \dot{m}_2$
  - Additionally for 1D flow ρ and V are constant over an area, so $\dot{m}_1 \dot{m}_2 = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$

Equation

Conservation of mass for a steady 1D flow:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

for a compressible flow, or

$$V_1 A_1 = V_2 A_2$$

for an incompressible flow

- Due to the 1D assumption, velocities would need to be given as average velocities

Energy Conservation (Euler and Bernoulli Equation)

- In fluids there are 3 types of energy: potential, kinetic, and pressure
- Assume steady and incompressible flow in an inviscid region (note inviscid implies laminar flow, since turbulence introduces frictional losses)
  - Friction will lead to energy losses, which is why this only works in inviscid regions
  - Therefore this is only an approximation and will not work in the boundary region (close to solid surface) and wake (turbulent) regions
- Consider a cylindrical differential element along the pathline/streamline
  - Use the coordinate system of s tangent to the streamline, s normal to the streamline
- For the s direction, $\sum F_s = ma_s$
  - $\sum F_s = W_s + \sum F_{p,s}$ (sum of weight and pressure forces)
  - $\sum F_s = -\rho g \, \mathrm{d}A \, \mathrm{d}s \sin\theta + p \, \mathrm{d}A - (p + \mathrm{d}p) \, \mathrm{d}A = -\rho g \, \mathrm{d}A \, \mathrm{d}z - \mathrm{d}p \, \mathrm{d}A$
  - $a_s = \dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}s}\dfrac{\mathrm{d}s}{\mathrm{d}t} = V\dfrac{\mathrm{d}V}{\mathrm{d}s}$
  - $ma_s = \rho \, \mathrm{d}A \, \mathrm{d}s \cdot V\dfrac{\mathrm{d}V}{\mathrm{d}s}$
  - Equating the two, we get Euler's equation: $V \, \mathrm{d}V + g \, \mathrm{d}z + \dfrac{1}{\rho} \, \mathrm{d}p = 0$

Equation

Euler's equation:

$$V \, \mathrm{d}V + g \, \mathrm{d}z + \frac{1}{\rho} \, \mathrm{d}p = 0$$

valid for steady, inviscid flow along a streamline (can be compressible)