Lecture 22, Oct 27, 2022

Forces on a Fluid

- Two categories:
  - Body forces: developed without physical contact, proportional to the fluid's mass
    - e.g. gravity
  - Surface forces: developed with physical contact at the surface of a fluid element
    - These don't have to be real surfaces
    - Can be broken down into components tangential (shear) or normal to the surface
    - Surface forces result in stresses

Definition

Normal stress is defined as

$$\sigma = \lim_{\delta A \to 0} \frac{\delta F_n}{A}$$

Shear stress is defined as

$$\tau = \lim_{\delta A \to 0} \frac{\delta F_t}{\delta A}$$

- Stress: Force per unit area
  - Stress at a point is defined as the limit as the area decreases to 0
  - Normal stress is defined as $\sigma = \lim_{\delta A \to 0} \frac{\delta F_n}{A}$
  - Shear stress is defined as $\tau = \lim_{\delta A \to 0} \frac{\delta F_t}{\delta A}$
  - Since there are multiple surfaces that can pass through a point, the stress at a point is described completely by specifying 3 stresses on mutually perpendicular planes through the point
- Double subscript notation for stress: τ_{xy} is a stress on the x plane (unit vector in the x direction) acting in the y direction
  - One normal stress σ_{xx}, two tangential stresses τ_{xy}, τ_{xz}, for every surface (in this case x)
  - This means there are 9 such stresses for every point! From these we can form a stress tensor to describe all stress components:
  - Stress tensor: $\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$

(Dynamic) Viscosity

- Recall shear stress in a solid is proportional to deformation angle $\delta\alpha$
- Since a fluid never stops deforming, shear stress is instead proportional to the rate of change in the deformation angle
  - $\tau \propto \frac{\mathrm{d}\alpha}{\mathrm{d}t}$
- In a parallel flow field with a velocity gradient (e.g. a boundary layer), the fluid undergoes shear forces as it deforms due to difference in velocity between layers
  - $\mathrm{d}\alpha = \frac{\mathrm{d}u}{\mathrm{d}y}\,\mathrm{d}t \implies \frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{\mathrm{d}u}{\mathrm{d}y}$
  - The rate of angular deformation of the fluid region is equal to the velocity gradient of the field
- Since $\tau \propto \frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{\mathrm{d}u}{\mathrm{d}y}$ for Newtonian fluids we can let $\tau = \mu\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \mu\frac{\mathrm{d}u}{\mathrm{d}y}$, where the proportionality constant μ is the *viscosity*

Definition

The viscosity of a Newtonian fluid is μ defined such that

$$\tau = \mu \frac{\mathrm{d}\alpha}{\mathrm{d}t} = \mu \frac{\mathrm{d}u}{\mathrm{d}y}$$

where $\frac{\mathrm{d}\alpha}{\mathrm{d}t}$ is the rate of angular deformation of the fluid element, equal to $\frac{\mathrm{d}u}{\mathrm{d}y}$, the velocity gradient in the fluid

- Newtonian fluids' viscosity are independent of the rate of deformation
  - On a graph of shear stress vs. velocity gradient, a Newtonian fluid starts at the origin and has constant slope
  - For non-Newtonian fluids, this graph is nonlinear and μ can vary with rate of deformation; some may even not start at the origin
    * Shear thinning: with higher rates of deformation, viscosity decreases (e.g. some paints, blood, cookie dough)
    * Shear thickening: with higher rates of deformation, viscosity increases (e.g. cornstarch + water)
    * Bingham plastic: acts like a solid until a certain shear stress, until a certain threshold after which it acts like a fluid (e.g. toothpaste)
      - Graph starts above zero
  - The *local viscosity/apparent viscosity* μ_{ap} is μ at the local conditions
- Viscosity has a strong dependence of temperature:
  - For liquids, viscosity *decreases* with temperature
    * Viscosity is caused by intermolecular forces; at higher temperatures molecules overcome these forces
  - For gases, viscosity *increases* with temperature
    * Viscosity is caused by molecular collisions; at higher temperatures molecules collide more
- "Viscosity" commonly refers to dynamic viscosity as opposed to kinematic viscosity

Definition

The kinematic viscosity $v = \frac{\mu}{\rho}$, where μ is the dynamic viscosity and ρ is the density

Compressibility

Definition

The bulk modulus is defined as $E_V = -\frac{1}{\cancel{V}} \frac{\mathrm{d}P}{\mathrm{d}\cancel{V}}$ where $\cancel{V}$ is the volume

- Bulk modulus measures compressibility; the larger it is, the less compressible the fluid
  - Note an increase in P causes a decrease in $\cancel{V}$, which is why there is a negative sign
  - Alternatively can be expressed as $E_V = \frac{1}{\rho} \frac{\mathrm{d}P}{\mathrm{d}\rho}$ (note there is no minus sign)
    * $m = \rho \cancel{V} \implies \mathrm{d}m = \rho \mathrm{d}\cancel{V} + \cancel{V} \mathrm{d}\rho = 0 \implies \frac{\mathrm{d}\rho}{\rho} = -\frac{\mathrm{d}\cancel{V}}{\cancel{V}}$
  - A truly incompressible flow has $E_V \to \infty$
    * For water it is 2.2×10^9 Pa, so it can be approximated as incompressible
    * Even though air is a lot more compressible, we can still assume it to be incompressible in a low speed flow