Lecture 2, Sep 9, 2022

More General Regions

Definition

- Type 1 region: $R = \{ (x, y) \mid a \leq x \leq b, g_1(x) \leq y \leq g_2(x) \}$
- Type 2 region: $R = \{ (x, y) \mid c \leq y \leq d, h_1(y) \leq x \leq h_2(y) \}$

Where g_1, g_2, h_1, h_2 are continuous

- Type 1 regions: hard boundaries in x, continuous varying boundaries in y
  - $V = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, \mathrm{d}y \, \mathrm{d}x$
- Type 2 regions: hard boundaries in y, continuous varying boundaries in x
  - $V = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \, \mathrm{d}x \, \mathrm{d}y$
- When the region of integration is neither, it can be cut up into type 1 and 2 regions
- When dealing with these, it's useful to first draw the planar region R
- Sometimes it can be much easier to integrate along one axis first, and then the other
  - e.g. $z = e^{x^2}$ over $y = x, 0 \leq x \leq 1$ is much easier to integrate along y first