Lecture 13, Oct 6, 2022

Line Integrals

- With a normal single integral we integrate over a coordinate axis (e.g. x axis); a line integral integrates over an arbitrary curve
- Defined just like a regular integral: $\int_C f(x,y)\,\mathrm{d}s = \lim_{\|P\|\to 0}\sum_{i=1}^{n} f(x_i^*, y_i^*)\Delta s_i$, where:
  - Divide the curve C into segments Δs_i
  - (x_i^*, y_i^*) is a point in the segment Δs_i
  - The norm of the partitioning $\|P\| = \max(\Delta s_i)$, i.e. the longest segment
- Geometrically, this is equivalent to the area of the surface between $f(x,y)$ and the x-y plane, along the path C (the "curtain" on curve C)
  - Note if we took $f(x,y) = 1$ we just get $\int_C \mathrm{d}s$ which is the arc length of C