Lecture 1, Sep 8, 2022

Partial Integration

> **Definition**
>
> A variable which is kept constant during integration is called a *parameter*

- The result of integration is a function of the parameter
- This type of integration is called a *partial integral*, since other variables other than the variable of integration are held constant
    - Opposite of partial differentiation
    - $\int_a^b f(x,y)\,\mathrm{d}x$ is a partial integral wrt x

Iterated Integrals

> **Theorem**
>
> Fubini's Theorem: $\displaystyle \int_a^b \int_c^d f(x,y)\,\mathrm{d}y\,\mathrm{d}x = \int_c^d \int_a^b f(x,y)\,\mathrm{d}x\,\mathrm{d}y$

- The order of integration for a double integral can be switched ("you can slice a region either way")
    - The counterpart is Clairaut's Theorem (symmetry of second partial derivatives)

> **Note**
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> In the special case where $f(x,y) = g(x)h(y)$,
>
> $$\int_c^d \int_a^b f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_a^b g(x)\,\mathrm{d}x \int_c^d h(y)\,\mathrm{d}y$$