# Lecture 5, Sep 20, 2021

## Projectile Motion in 1D

- "Projectile": any object launched with some initial velocity $v_0$; modelled by $x_f = x_i + v_i t_f + \frac{1}{2}at_f^2$
- A projectile launched upwards with $v_i$ and a projectile launched downwards with $-v_i$ have the same downward speed when the projectile passes through the starting point
  - $x_f = x_i \implies v_i t + \frac{1}{2}at^2 = 0$ and $v(t) = v_i - gt \implies t = \frac{v(t) - v_i}{-g} = \frac{v_i - v}{g} \implies 0 = v_i\left(\frac{v_i - v}{g}\right) - \frac{1}{2}g\left(\frac{v_i - v}{g}\right)^2 = \left(\frac{v_i - v}{g}\right)\left[v_i - \frac{1}{2}g\left(\frac{v_i - v}{g}\right)\right] = \left(\frac{v_i - v}{g}\right)\left[v_i - \frac{1}{2}(v_i - v)\right]$, zero when $v = -v_i$ or $v = v_i$

## Inclined Planes and Free Fall

- Galileo observed that the ratio $\frac{x_i}{t_i^2}$ was constant; i.e. the position is proportional to time squared, when the object is rolling down an inclined plane
  - This ratio is a function of theta: $ax = g\sin\theta$
- When the plane is at 90°, the object is in free fall

## Instantaneous Acceleration

- $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) \equiv \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$
- The instantaneous acceleration is the "curvature" of the position function (related? equal to? the actual curvature $\kappa = \frac{1}{R}$)