

# Lecture 3, Sep 15, 2021

## Motion in 1D

### The Calculus of Motion

- The calculus hierarchy:  $x(t) \xleftrightarrow[\int dt]{\frac{d}{dt}} v(t) \xleftrightarrow[\int dt]{\frac{d}{dt}} a(t)$ 
  - Integration only tells you how much the functions changed, not where they started
- Essential calculus:  $\frac{d}{dt} t^n = n t^{n-1} \implies \int n t^{n-1} dt = t^n + C \implies \int t^n dt = \frac{1}{n+1} t^{n+1}$
- Example:  $x(t) = 2[m] + 3 [m/s^3] t^3$  (square brackets denote units)
  - $v(t) = \frac{d}{dt} (2 + 3t^3) = 0 + 9t^2$
- Example:  $v(t) = 3 [m/s]$  find  $x(t)$  for  $x(0) = 2 [m]$ 
  - $x(0)$  is the constant of integration
- When differentiating by time, a unit of time is introduced in the denominator; e.g.  $\frac{d}{dt} [m] = [m/s]$

### Units of Motion

Quantity	Symbol	Units
time	$t$	T
position	$x$ or $\vec{r}$ in multiple dimensions	L
velocity	$v$ or $\vec{v}$ in multiple dimensions	L/T
acceleration	$a$ or $\vec{a}$ in multiple dimensions	L/T <sup>2</sup>
speed	$ v $ or $\ \vec{v}\ $ in multiple dimensions	L/T