Lecture 29, Dec 1, 2021

Simple Harmonic Motion

- Simplest example of an oscillator is a mass on a spring
- The restoring force is $-kx$ so $F = ma = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx \implies \frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{k}{m}x = 0$
  - The solution to this DE is a sinusoid $A\cos(\omega t + \phi)$
  - Substituting this back in we get $\omega = \sqrt{\frac{k}{m}}$
- Simple harmonic motion is like looking at uniform circular motion in one axis
- When the system oscillates, there is a constant conversion between kinetic and potential energy
  - At the extremes all the energy is contained in the spring as potential energy
  - When the system passes through equilibrium all the energy is kinetic
  - In-between there's a mix of the two
- Simple harmonic oscillators have the same period regardless of amplitude; no matter how far you pull the spring when you start it, it will still take the same time to complete a full cycle (ω does not depend on A)
- $\begin{cases} x(t) = A\cos(\omega t + \phi) \\ v(t) = -A\omega\sin(\omega t + \phi) \\ a(t) = -A\omega^2\cos(\omega t + \phi \end{cases}$
  - Maximum displacement is A
  - Maximum velocity is $A\omega$
  - Maximum acceleration is $A\omega^2$