Lecture 22, Nov 3, 2021

Two Dimensional Kinematics

- Velocity and acceleration in 2 dimensions can be decomposed into components
- The velocity $\vec{v}$ is always tangent to the trajectory
- Acceleration $\vec{a}$ can be decomposed into two components, one tangent to the trajectory (same direction as $\vec{v}$) which only accelerates the object, and one perpendicular to the trajectory (orthogonal to $\vec{v}$) which only changes the object's direction
  - This is true instantaneously; over a time interval, as the perpendicular component of acceleration changes the direction of velocity, more and more of the acceleration will go to increasing the velocity rather than changing its direction
- In multiple dimensions position $\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$, $\vec{v}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}(t) = \begin{bmatrix} \frac{\mathrm{d}x}{\mathrm{d}t} \\ \frac{\mathrm{d}y}{\mathrm{d}t} \\ \frac{\mathrm{d}z}{\mathrm{d}t} \end{bmatrix}$ and so on for acceleration
  - The speed is the magnitude of $\vec{v}$
- When we differentiate/integrate to get from one to the other, we can do so for each component separately
- We typically want to find the simplest coordinate system, usually one that has no motion in one or more dimensions, in orders simplify the problem

Projectile Motion

- The special case of constant acceleration where $\vec{a} = -g\hat{\jmath}$
- Horizontal component of velocity is constant (ignoring air friction)
- Since the slope of the curve is $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v_y}{v_x}$
- $\vec{r} = \frac{1}{2}\vec{a}t^2 + \vec{v}t \implies \begin{cases} x = v_x t = v_0 \cos\theta t \\ y = -\frac{1}{2}at^2 + v_y t = -\frac{1}{2}gt^2 + v_0 \sin\theta t \end{cases}$; eliminate t by substituting $t = \frac{x}{v_0 \cos\theta}$ to get $y = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$
- To find the range, we solve for when $y = 0 \implies \frac{gx^2}{2v_0^2\cos^2\theta} = x\tan\theta \implies x = \frac{v_0^2}{g}\sin(2\theta)$ (use double angle identity $2\sin\theta\cos\theta = \sin(2\theta)$)
  - In order to maximize range $\sin(2\theta) = 1 \implies 2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4} = 45°$
- The max flight time only depends on v_y, and occurs when $y = 0 \implies v_0 \sin\theta t = \frac{1}{2}gt^2 \implies t = \frac{2v_0\sin\theta}{g}$, so the object should be thrown straight up

Decomposing Forces

- Example: Pulling someone on a swing; the force needed grows larger as the angle of the swing increases
- We can decompose the force being applied by the swing pivot into two components, one perpendicular the ground and one parallel to the ground
- As you pull the person back, the angle increases and the lateral component of the swing pivot force grows, which needs to be balanced out by the applied force
  - The vertical component of the swing pivot force is balanced by gravity; the diagonal force of the pivot has a vertical component to counteract gravity, and as a side effect this produces a horizontal component that must be balanced out by the applied force