Lecture 21, Nov 1, 2021

Work Done on Springs

- The reaction force of a spring is $k(x - x_0)$ by Hooke's law and if we set the relaxed length to 0, we get $F = -kx$; since this force is equal and opposite to the applied force, the applied force is kx
- The work done by the hand compressing the spring is $\displaystyle\int_0^{x_b} kx \, \mathrm{d}x = \frac{1}{2}kx_b^2$
- The potential energy of the spring is a quadratic function
- Springs are an example of *stable* equilibrium; at $x = x_0$, the spring will actively resist any force that pushes it away from equilibrium
  - *Unstable equilibrium* is like a ball on a hill; if it is pushed even slightly, it will tend towards some other state instead of returning to the same state

Power

- Power is the rate at which energy changes, $\dfrac{\mathrm{d}E}{\mathrm{d}t}$ or $\dfrac{\Delta E}{\Delta t}$ if the rate is constant; alternatively $P = Fv$
- Power is measured in $\mathrm{J/s} = \mathrm{W}$

Two-Dimensional Motion

- In two dimensions quantities are represented as two-dimensional vectors $\begin{bmatrix} x \\ y \end{bmatrix}$
- The vectors can be in any coordinate system, e.g. the axes can be rotated or stretched