Lecture 17, Oct 21, 2021

Newtons Laws From Momentum Conservation

- Newton's first law (isolated objects stay at rest/in motion) is a result of momentum being conserved in an isolated system
- Newton's second law ($F = ma$, definition of force) is a result of differentiating momentum $\dfrac{\mathrm{d}p}{\mathrm{d}t}$
- Newton's third law (every force has an equal and opposite reaction) is a result of differentiating $\Delta p_1 = -\Delta p_2$

Superposition of Forces

- Forces can be superimposed; the result of several forces on an object is an acceleration equal to the sum of the accelerations caused by the individual accelerations

Springs and Tension

- Springs extend to generate forces and pull loads into equilibrium
- The force generated by a spring as the result of a displacement is proportional to the magnitude of displacement (Hooke's law)
- This is only valid for a limited range of extensions and contractions

Impulse

- $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t} \implies m\vec{a} = m\dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\Delta\vec{p}}{\Delta t}$
- We define $\Delta\vec{p} = \vec{J}$, the *impulse* (change in momentum)
- Since $\sum\vec{F} = m\vec{a}$, so $\vec{J} = \Delta t\sum\vec{F}$
- If force varies over time then $\vec{J} = \displaystyle\int_{t_i}^{t_f}\sum\vec{F}(t)\,\mathrm{d}t$
- As area under the acceleration curve is change in velocity, area under the force curve is impulse

Example: Tennis Racket Launching Ball

- Suppose a tennis racket hits a ball (0.20kg) and immediately after the collision the ball has an acceleration of $9g$ upwards, how much force needs to be applied by the racket?
  - $\sum F = F_{Eb}^{G} + F_{rb}^{C} = m_b a \implies F_{rb}^{C} - m_b g = 9m_b g \implies F_{rb}^{C} = 10m_b g$
    * Note superscripts are used to indicate type of force (**G**ravity, **C**ontact), and subscripts are used to denote the objects (**E**nvironment on ball, **r**acket on ball)
  - Substitute $m_b = 0.20\text{kg} \implies F_{rb}^{C} = 20\text{N}$