Lecture 10, Sep 30, 2021

Coefficient of Restitution

- $e = 1 + \frac{2\Delta E_{int}}{m_1 \Delta v_1 v_{2,1i}}$ (from $K_i = \Delta E_{int} + K_f$)
  - $v_{2,1i}$ is always less than zero
    - If the two carts are heading towards each other then $v_{2,1i} = v_{1i} - v_{2i}$ is positive, but Δv_1 is negative since cart 1 slows down
  - Since the term on the right is always less than zero, any $\Delta E_{int} \neq 0$ will result in a e less than 1
  - $e \neq 1$ is a result of change in internal energy $\Delta E_{int} \neq 0$
  - When $\Delta E_{int} < 0$, $e > 1$, and this is an explosive separation (energy is introduced into the system)

Explosive Separations

- Cases where $e > 1$, kinetic energy is introduced (from the internal energy of the explosive/spring/etc)

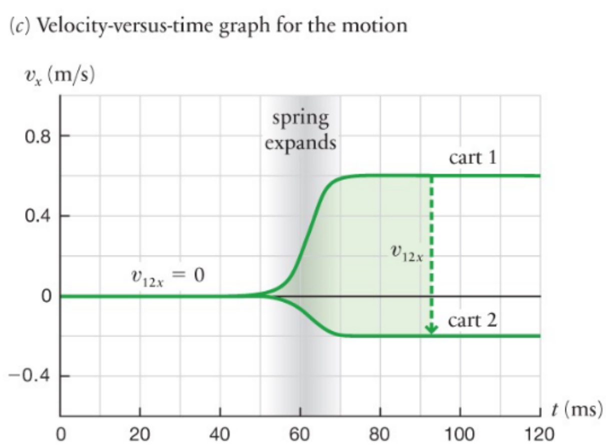

Figure 1: explosive separation

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- Solve for v_{1f} and v_{2f} with $v_{1i} = v_{2i} = 0$
  - $E_{spring} = -\Delta E_{int}$
  - $$\Delta K + \Delta E_{int} = 0$$
    $$\implies K_f - K_i + \Delta E_{int} = 0$$
    $$\implies K_f = -\Delta E_{int}$$
    $$\implies K_f = E_{spring}$$
    $$\implies \frac{1}{2}(m_1 v_{1f}^2 + m_2 v_{2f}^2) = E_{spring}$$
  - $$m_1 v_{1f} + m_2 v_2 f = 0$$
    $$\implies v_{2f} = \frac{-m_1}{m_2} v_{1f}$$
    $$\implies \frac{1}{2}\left(m_1 v_{1f}^2 + m_2 \left(\frac{-m_1}{m_2} v_{1f}\right)^2\right) = E_{spring}$$
    $$\implies v_{1f}^2 = \frac{2E_{spring}}{m_1 + \frac{m_1^2}{m_2}}$$

Reference Frames and Relativity

- Note: The relativity we're talking about assumes $v \ll c$
- How does motion vary from different perspectives? Are energy, momentum, etc conserved?
- Right now limited to observers with constant velocity difference
- $\vec{v}_{A,B}$ is the velocity of A as observed by B
- Define all speeds relative to an observer: $\vec{v}_{o,r} = \frac{\mathrm{d}}{\mathrm{d}t}(\vec{r}_o - \vec{r}_r) = \frac{\mathrm{d}}{\mathrm{d}t}\Delta\vec{r}_{r\to o}$ (reference r, object o)
- Physics that work regardless of perspective:
  - Velocity of objects ✗
  - Momentum of objects ✗(because velocities are different)
  - Total kinetic energy ✗(because velocities are different)
  - Where things happen ✗
  - When things happen ✓(but not in special relativity)
  - Relative velocities ✓
  - Change in momentum ✓
  - Loss of kinetic energy ✓(because they agree on Δv)
  - Increase of internal energy ✓
  - Conservation of momentum ✓
  - Conservation of energy ✓
- Change in kinetic energy holds across perspectives
  - $k_1 = \frac{1}{2}m_1 v_1^2, k_2 = \frac{1}{2}m_2 v_2^2$
  - Observers will agree on Δv_1 and Δv_2
  - Frame A:
    * $\Delta K_A = K_{A,f} - K_{A,i} = \frac{1}{2}m_1(v_{A1f}^2 - v_{A1i}^2)$