# Lecture 3, Sep 15, 2021

## Review

1. Absolute value $|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$
   - Easier to work with: $|a| = \sqrt{a^2}$
     - Note: Square roots are zero or positive or does not exist in the reals; e.g. $\sqrt{4}$ is **only** 2, not $\pm 2$
     - Different question: What $x$ satisfies $x^2 - 4 = 0$? This is a *different question* and is $\pm 2$
2. Intervals of $x$ values
   1. $x \in [a, b]$ – closed interval ($a \leq x \leq b$), which is a set of numbers; filled dots on the number line
   2. $x \in (a, b)$ – open interval ($a < x < b$); open dots on the number line
   3. $x \in [a, b)$, $x \in (a, b]$ – mixed open/close intervals
   4. $x \in (-\infty, b]$ – all $x \leq b$
      - Infinity is only okay to use inside an expression when the whole expression is defined, because $\infty$ is not a number
      - Closed brackets on infinity, e.g. $[a, \infty]$, is undefined because there is no useful definition for it
3. Functions: Given two sets of numbers $x$-set and $y$-set, a function is a rule that we specify that relates each $x$ to *one* $y$ value (very general)
   - $x$ *independent* variable, $y$ *dependent* variable; $x$ could be anything as long as it produces a defined $y$
   - Note the asymmetry: each $x$ can only relate to one $y$, but this requirement does not exist for $y$
   - Example: A table of $x$ and $y$ values could be a function
   - The span of the set of $x$ values is the *domain*; the span of the set of $y$ values is the *range*
   - Functions do not have to be **well-behaved**: e.g. $f(x) = \begin{cases} 10x + 5 & x \neq 0 \\ DNE & x = 0 \end{cases}$ is a perfectly good function; so is $f(x) = \begin{cases} 1 & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}$
4. Trigonometric functions:
   - Prefer algebraic definitions over geometric definitions
   - Prefer angles to be in radians where $2\pi$rad is a full revolution (geometric definition of $\pi$)
     - For radius $r$ the arc length corresponding to angle $x$ is just $rx$ (check: if $x = 2\pi$ then $rx = 2\pi r$)
5. Composition of functions: $f(g(x))$
   - e.g. if $f(x) = 3x^2 + 2$ and $g(x) = \sin x$ then $f(g(x)) = 3\sin^2 x + 2$ and $g(f(x)) = \sin(3x^2 + 2)$
6. Increasing/decreasing functions
   - Given $x_1 > x_2$ for any two values of $x_1$ and $x_2$ in some interval, if $f(x_1) > f(x_2)$, define $f(x)$ to be **increasing** on this interval; if $f(x_1) < f(x_2)$, define $f(x)$ to be **decreasing** on this interval
   - Example: Prove $x^2 + 2$ is increasing for $x > 0$
     - Take any $x_1 > x_2 > 0$
     - $0 < x_1^2 < x_1 x_2$ and $0 < x_1 x_2 < x_2^2 \implies x_1^2 < x_2^2 \implies x_1^2 + 2 < x_2^2 + 2 \implies f(x_1) < f(x_2)$
7. Even/odd functions
   - $f(x) = f(-x)$ is an even function; $f(x) = -f(-x)$ is an odd function
8. Basic Arithmetic Theorem (BAT1)
   - Prove a true equality: e.g. $1 + 2 = 6 - 3$
     - You can add, subtract, multiply, and divide both sides of an equality by the same factor and get another true equality
     - This is a theorem and can be proven
   - Prove a true inequality: e.g. $3 < 5$
     - Get another true inequality by applying the same operations, **except** if multiplication or division is involved *and* the factor is negative, then the direction of the inequality changes
     - Also a theorem that can be proved using axioms