Lecture 22, Nov 1, 2021

Solids of Revolution

- A solid of revolution is a solid obtained by revolving a region about an axis, usually (but doesn't have to be) the x axis
- The area of each slice is then $A_i = \pi(f(x))^2$ so $V = \int_a^b \pi(f(x))^2 \, \mathrm{d}x$, known as the disk method
  - Example: Volume of cone with height h and radius r
    - $y = \frac{r}{h}x \implies V = \int_0^h \pi \left(\frac{rx}{h}\right)^2 \, \mathrm{d}x = \frac{1}{3}\pi r^2 h$
- Sometimes the curve might go below the x axis, but this is fixed by the square
- If the region is bounded by two curves, when rotated about an axis, each slice would be a ring (washer), with area equal to the difference of two circles
- $A_i = \pi\left[(f(x))^2 - (g(x))^2\right] \implies V = \int_a^b \pi\left[(f(x))^2 - (g(x))^2\right] \mathrm{d}x$, known as the washer method
- If the axis is parallel but not equal to the x axis, then we need to subtract an offset $V = \int_a^b \pi(f(x)-k)^2 \, \mathrm{d}x$ for the disk method, or $V = \int_a^b \pi\left|(f(x)-k)^2 - (g(x)-k)^2\right| \mathrm{d}x$ for the washer method (note the added absolute value)
  - Note the constraint $k > f, g$ or $k < f, g$; the line cannot cross the region itself because then the region would be rotating into itself
- Example: The region between $y = x^2$ and $y = x$ about $y = -2$
  - The two curves intersect at 0 and 1
  - $V = \int_0^1 \pi\left[(x+2)^2 - (x^2+2)^2\right] \mathrm{d}x$