Lecture 9, Oct 13, 2021

Proving Trig Identities Using Matrices

- Rotation matrix: $T_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- Define the composition $T_\theta(T_\theta(\vec{v})) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$
- Since $T_\theta(T_\theta(\vec{v})) = T_{2\theta} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ but $T_\theta(T_\theta(\vec{v})) = \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$, we can conclude $\begin{cases} \sin 2\theta = 2\sin\theta\cos\theta \\ \cos 2\theta = \sin^2\theta - \cos^2\theta \end{cases}$

Eigenvalues & Eigenvectors

- Certain vectors do not change direction (remain parallel) when transformed by a linear transformation; they are the *eigenvectors* of that transformation, and the amount they are scaled by are the *eigenvalues*
- $A\vec{v} = \lambda\vec{v}$, where $\vec{v}$ is an eigenvector and λ is an eigenvalue
- Example: For a transformation that projects every vector onto a fixed vector, every vector parallel to the fixed vector is an eigenvector of the transformation with eigenvalue 1; every vector orthogonal to it is an eigenvector of the transformation with eigenvalue 0
- The number of eigenvalues (note: these can be complex) is always the same as the dimension of the matrix (note: nonsquare matrices don't have eigenvalues)
- To find eigenvectors, note $A\vec{v} = \lambda\vec{v} \implies A\vec{v} - \lambda\vec{v} = 0 \implies A\vec{v} - \lambda I\vec{v} = 0 \implies (A - \lambda I)\vec{v} = 0$, so the eigenvectors are the null space of $A - \lambda I$
  - Note that this null space is only nontrivial if $\det(A - \lambda I) = 0$, so we can find eigenvalues by finding the values of λ that make $\det(A - \lambda I) = 0$