Lecture 5, Sep 27, 2021

Lines and Planes in 3D

- Lines in $\mathbb{R}^3$:
  - Line in $\mathbb{R}^3$ through the origin: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = c\vec{d}$ where $\vec{d}$ is a direction vector on the line
  - General line in $\mathbb{R}^3$ not through the origin: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = p_0 + c\vec{d}$ where p_0 is a known point on the line
  - Both have only one free parameter c, which corresponds with the fact that the line is 1-dimensional
  - To convert $y = mx + b$ to vector form: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix} + c \begin{bmatrix} 1 \\ m \end{bmatrix}$
    * Our known point: $\begin{bmatrix} 0 \\ b \end{bmatrix}$
    * The direction vector: $\begin{bmatrix} 1 \\ m \end{bmatrix}$: as x increases by 1, y increase by m
- Planes in $\mathbb{R}^3$:
  - Plane in $\mathbb{R}^3$ through the origin: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = c\vec{d_1} + d\vec{d_2}$ where $\vec{d_1}$ and $\vec{d_2}$ are two independent vectors parallel to the plane
  - General plane in $\mathbb{R}^3$ not through the origin: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = p_0 + c\vec{d_1} + d\vec{d_2}$ where p_0 is a known point on the plane
  - Another way to represent planes is with a normal vector $\vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$; while there are many vectors parallel to the plane, there are only two directions that are normal to the plane
    * Since $\vec{n}$ is orthogonal, any vector $\overrightarrow{p_0p}$ that goes from a known point on the plane p_0 to another point on the plane p_1 is orthogonal to it
    * $0 = \overrightarrow{p_0p} \cdot \vec{n} = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} \implies a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \iff ax + by + cz = d$ where $d = ax_0 + by_0 + cz_0$
    * This equation is sometimes referred to as the scalar equation of the plane
  - To go from vector equation to scalar equation, take the cross product to get a normal vector, substitute in a, b, and c and solve for d
  - To go from a scalar equation to a vector equation, find 3 non-collinear points and get two vectors and use any point as the known point