Lecture 22, Dec 8, 2021

Computing LU Decompositions

- There is a simpler procedure for doing LU decompositions:
    1. Reduce matrix A to upper-triangular U by elimination; keep track of the multipliers used to introduce the leading 1s and zeroes below
    2. Along the main diagonal of L, put the reciprocal of the multiplier that introduced the leading 1 in U in that position
    3. Below the main diagonal of L, put the negative of the multiplier used to introduce the 0 in U in that position

Review: The 4 Central Problems of Linear Algebra

1. Solving linear systems $A\vec{x} = \vec{b}$ where $m = n$
    - Arises from mathematical models of static engineering systems
2. Least squares $A\vec{x} = \vec{b}$ where $m \neq n$
    - Arises when fitting mathematical models to real data with noise
3. Eigenvalues $A\vec{x} = \lambda \vec{x}$ for $m = n$
    - Arises when analyzing and designing dynamic systems (systems of differential equations, control systems)
4. The singular value decomposition $A\vec{v} = \sigma \vec{u}$ for $m \neq n$ (analogue of eigenvalues for nonsquare matrices)
    - Arises with data and image compression problems