Lecture 2, Sep 15, 2021

Vector Properties

- Suppose you have 3 vectors $\vec{v}, \vec{w}, \vec{z}$ in $\mathbb{R}^3$, for scalars c, d, e
  - The picture of all linear combinations $c\vec{v}$ is a line, $c\vec{v} + d\vec{w}$ is a plane, $c\vec{v} + d\vec{w} + e\vec{z}$ is all of $\mathbb{R}^3$ assuming independence
- Vectors are linearly independent when they cannot be expressed as linear combinations of others
- Vector magnitude $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots}$
  - Properties of magnitudes (and all norms in general):
    1. $\|c\vec{v}\| = |c|\|\vec{v}\|$
    2. $\|\vec{v}\| \iff \vec{v} = \vec{0}$
- A unit vector $\hat{v}$ is any vector with length 1
  - In $\mathbb{R}^3$ the well known unit vectors are $\hat{\imath}, \hat{\jmath}, \hat{k}$ but there are an infinite number of them
  - Vectors can be made into unit vectors (normalized) by dividing by their magnitude: $\hat{v} = \dfrac{\vec{v}}{\|\vec{v}\|}$

Distance Between Points

- To calculate the distance between two points, we first need to establish a coordinate system, with an origin and axis directions (basis vectors?)
- Make vectors $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ where O is the origin, and $\overrightarrow{P_1P_2}$, which we want the magnitude of
- $\overrightarrow{OP_1} + \overrightarrow{P_1P_2} = \overrightarrow{OP_2} \implies \overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$ then find $\left\|\overrightarrow{P_1P_2}\right\|$

Dot Product

- Definition: The *dot product* $\vec{v} \cdot \vec{w} = \displaystyle\sum_{i=1}^{n} v_i w_i$ (sum of products of each corresponding pair); note the dot product of vectors is a scalar, so it is sometimes referred to as the *scalar product*