Lecture 18, Nov 22, 2021

Initial Value Problems

- An IVP is a differential equation attached with initial conditions that allow for a particular solution
- General problem: Given $\dfrac{\mathrm{d}y}{\mathrm{d}t} = y'(t) = cy(t)$ where c is a scalar and $y(0) = y_0$ solve for $y(t)$ for $t > 0$
- We begin with $y(t) = y_0 e^{ct}$, so $\dfrac{\mathrm{d}y}{\mathrm{d}t} = cy_0 e^{ct} = cy(t)$ so it satisfies the DE; since $y(0) = y_0 e^0 = y_0$ it also satisfies the initial conditions
- Generalizing this we have given $y'(t) = f(t, y(t))$ and $y(0) = y_0$
- In this course we will focus on the numerical approach to solving this equation
- We start with the initial value $y(0) = y_0$, and now we can evaluate $f(t, y(t))$ to obtain a slope and take a step forward by some amount Δt and use the first derivative information to find the y value for this new point, and repeat until the desired t is reached

Reducing the Error

- To reduce the error we can try to get a better slope estimate
- We can take $S = \dfrac{1}{2}(S_L + S_R)$ where $S_L = f(t_n, y_n)$ and $S_R = f(t_{n+1}, y_{n+1})$, with y_{n+1} estimated using the first method, i.e. take slope to be the average of the two points