Lecture 13, Oct 27, 2021

Gaussian Elimination

- Three outcomes are possible with Gaussian elimination:
    1. Unique solution: When every variable in the RNF is a leading variable
    2. Infinitely many solutions: When there is at least 1 free variable
    3. No solutions: When you get a row of all 0s but the last entry is nonzero
- Cases 1 and 2 are *consistent* systems (at least 1 solution); 3 is an *inconsistent* system

Rank – The True Size of a Matrix

- Suppose $A\vec{x} = \vec{b}$ and A is $m \times n$, define rank(A) to be the number of leading 1s in the RNF, or equivalently the number of linearly independent rows or columns (row rank equals column rank)
- The RNF is unique even though the Gaussian elimination steps can be done in any order
- The system only has a unique solution when rank(A) $= m = n$; when $r < m$ or $r < n$ there are no solutions or infinite solutions