Lecture 12, Oct 25, 2021

Reduced Normal Form/Reduced Row Echelon Form

- The leading variable (one per equation) only appearing in one row makes the system easy to solve because the leading variables can be expressed in terms of free variables
- Define the *augmented matrix* $[A|\vec{b}]$ as a shorthand for the system $A\vec{x} = \vec{b}$
- The form of the augmented matrix that makes it easy to solve is called the *reduced normal form* (RNF)
  - Matrices in RNF has the following properties:
    1. The first nonzero entry in each row is 1
    2. The other entries in the columns containing the leading 1s (above and below) are 0
    3. The leading 1s move to the right as we move down the rows
    4. Any and all zero rows are at the bottom
- If the system is in RNF, we can proceed directly to solve it, but if it is not, we can use Gaussian elimination to bring it to RNF $[R|\vec{d}]$, an *equivalent* linear system that has the same solution as the original system $[A|\vec{b}]$

Gaussian Elimination

- By swapping rows, multiplying rows by nonzero scalars, and adding multiples of one row to another, we change the system of equations but don't change the solution
- If we do this algorithmically we can bring the matrix to RNF and make it easy to solve
- For the purposes of just solving the system, we usually don't have to go all the way; stop when the bottom left corner is all zeroes and do the rest using back substitution
- The allowed operations swapping rows, multiplying by nonzero scalars, and adding multiples of one row to another are called *elementary row operations*