Lecture 11, Oct 20, 2021

Solving Systems of Equations

- Example: Solving $ax_1 + bx_2 + cx_3 = d$
  - Declare x_1 to be the leading variable and x_2, x_3 as the free variables
  - Solve for x_1 in terms of x_2 and x_3: $x_1 = \alpha x_2 + \beta x_3 + \gamma$
  - Use the additional equations $x_2 = x_2$ and $x_3 = x_3$ to write the solution in vector form
  - $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \gamma \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}$
  - Since x_2 and x_3 are free variables that can be freely assigned, they are arbitrary scalars, which means any solution can be found with $\begin{bmatrix} \gamma \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}$ for some arbitrary scalars c and d
  - This is basically converting a scalar equation of the plane into a vector equation of the plane
  - In general, express the leading variables in terms of the free variables (number of leading variables equals number of equations), and then you can write the vector solution form
- Solving a system of equations corresponds to solving $A\vec{x} = \vec{b}$
- In the row picture, the solution represents the intersection of m hyperplanes of $n-1$ dimensions in $\mathbb{R}^n$
- In the column picture, the solution represents all linear combinations of the columns of the matrix that gives $\vec{b}$