Lecture 10, Oct 18, 2021

Finding Eigenvectors Given Eigenvalues

- Suppose we have λ, how do we solve $(A - \lambda I)\vec{v} = \vec{0}$?
- Multiplying it out gets us a system, but some equations are not independent; we are left with equations relating the components of $\vec{v}$
- This allows us to express all the other components all in terms of one component, and allows us to write the vector $\vec{v}$ as the product of a scalar (one of the components) and a vector, which an eigenvector

Inverses

- Suppose $\vec{u} = T(\vec{w})$, given $\vec{u}$ and T, can we find $\vec{w}$?
  - If T is linear, $\vec{u} = M_T\vec{w}$
  - To solve the problem we want to find another matrix N such that $NM_T = I$, so that $N\vec{u} = NM_T\vec{w} = I\vec{w} = \vec{w}$
  - N is the inverse of M_T, M_T^{-1}
- For the 2×2 case where $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $M^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Determinants

- $ad - bc$ is the *determinant* of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- The determinant determines whether the inverse exists; the inverse only exists when $\det A \neq 0$
- $(M - \lambda I)$ is not invertible; proof:
  - Assume it is invertible, this would mean N exists such that $N(M - \lambda I) = I$
  - Therefore, $N(M - \lambda I)\vec{v} = N\vec{0} \implies I\vec{v} = \vec{0} \implies \vec{v} = \vec{0}$
  - Therefore if $M - \lambda I$ is invertible then the only $\vec{v}$ that satisfies the equation is $\vec{v} = \vec{0}$
- Therefore, the condition for finding eigenvalues is to solve for when $\det(M - \lambda I) = 0$