Lecture 1, Sep 13, 2021

Introduction to Vectors

- Vectors: direction, magnitude, units; scalars: magnitude, sign
- Core of linear algebra involves 2 operations: Adding vectors and scaling vectors
- Course notation: vector from p (tail) to q (head) is expressed as $\vec{v} = \overrightarrow{PQ} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ (arrowhead is complete)

- Row vectors are not equal to column vectors: $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \neq \begin{bmatrix} v_1 & v_2 \end{bmatrix}$

• Standard position for vectors is tail at the origin

Vector Operations

Adding vectors: \$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix}\$

Geometrically this puts the tail of one vector at the head of another

Scaling vectors: \$c \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} cv_1 + w_1 \\ v_2 + w_2 \end{bmatrix}\$

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This makes the vector longer or shorter/flips it

- When one vector is a scalar multiple of another, they're parallel $\vec{v} \parallel \vec{w}$
- The zero vector is $\vec{v} \vec{v} = \vec{0}$
- A linear combination of two vectors \vec{v} and \vec{w} is $c\vec{v} + d\vec{w}$ for any c and d

Vector Properties

1. $\vec{v} + \vec{w} = \vec{w} + \vec{v}$ (commutative)