Lecture 9, Sep 28, 2021

Relationship Between Moment and Angular Acceleration

- If we apply a pure moment M, what is the angular acceleration of the mass?
  - If the lever arm is y then $M = Fy$
  - The angular acceleration is related to the translational acceleration: $a = \alpha y$
  - If we combine them with $F = ma$: $M = Fy = (m\alpha y)y = m\alpha y^2 = (my^2)\alpha$
  - The my^2 term is the angular equivalent of mass, the *moment of inertia* I_m
- $M = I_m a$ is the rotational analogue of Newtons second law, and I_m has units of mass times length squared

Calculating the Moment of Inertia

- To determine I_m for non-point masses, we can break the object into smaller pieces:

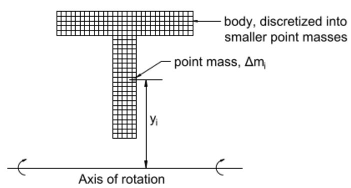

Figure 1: breaking the object into smaller pieces

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  - $I_m = \sum I_{m,i} = \sum \Delta m_i y_i^2$
- We can get the exact moment of inertia by taking an integral: $I_m = \int_M y^2 \,\mathrm{d}m$ where M is the entire mass
- For a 2-dimensional object with uniform density ρ this reduces to $\rho \int_A y^2 \,\mathrm{d}A$
- The integral term $\int_A y^2 \,\mathrm{d}A$ is known as the *second moment of area* I, with dimensions of length to the power of 4

Properties and Physical Interpretation of the Moment of Inertia

- From the formula we can see that I depends on the axis of rotation, and masses further from the axis of rotation contribute more to the moment of inertia

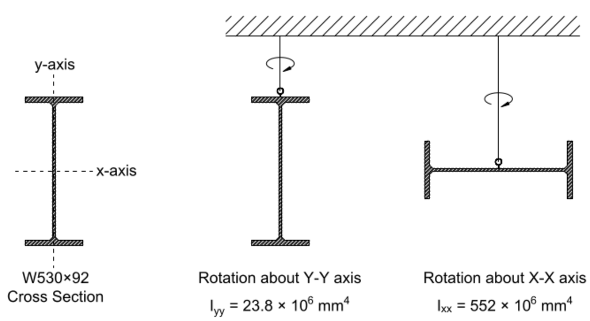

Figure 2: moment of inertia for I-beam

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- Example: W530 $\times$ 92 I-beam (530mm nominal height, 92kg/m weight); the moment of inertia about the y axis is much lower because the masses are closer to the axis

Example Calculation

- Example: Calculate the second moment of area for a rectangle rotating about its middle axis
- $\mathrm{d}A = b\,\mathrm{d}y$ where b is the width of the rectangle
- $I = \int_{-\frac{h}{2}}^{\frac{h}{2}} by^2 \,\mathrm{d}y = \left[\frac{1}{3}by^3\right]_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{bh^3}{12}$