Lecture 8, Sep 28, 2021

- Types of oscillations:
  1. Mechanical
     - Springs – we'll be looking at this for this lecture
     - Earthquakes
     - Sound transmission
     - Machine vibration
  2. Electrical
     - Power generation and transmission
     - Electromagnetic (communications)
     - Photonics

Dynamic Equilibrium

- Oscillating systems are not in static equilibrium, so we cannot use our usual methods
- We can apply D'Alembert's Principle: Convert dynamic equilibrium ($a \neq 0$) to equivalent static equilibrium by introducing a fictitious *inertial force*
  - This inertial force is $F_i = ma$ and acts in the opposite direction of acceleration
  - Think of it as inertia resisting acceleration
  - By doing this we can convert it into a static system where $\sum F = 0$, and we no longer have to consider time
  - The inertial force is fictitious but consistent with physics
  - $\sum F = ma \implies \sum F - ma = 0 \implies \sum F - F_i = 0$ if $F_i = ma$ is considered a force
  - Example: centrifugal force is not a real force but we still feel it

Free Vibration Without Gravity in One Dimension

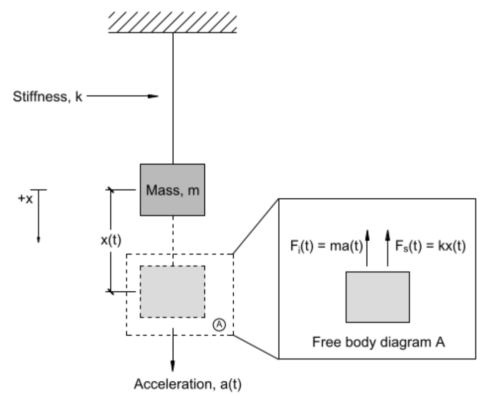

Figure 1: Free vibration

- Consider a mass attached to a spring; ignoring all resistance, if the mass is pulled down and then released, the system is in free vibration
- The system can be modelled by a second-order linear homogeneous DE: $ma(t) + kx(t) = 0 \implies mx''(t) + kx(t) = 0$
- This has solution $x(t) = A\sin(\omega_n t + \phi)$, where A is amplitude, ω_n is the natural angular frequency, and ϕ is the phase delay
  - If we substitute back this solution, we get $\omega_n = \sqrt{\dfrac{k}{m}}$
  - This shows that the frequency is dependent only on the mass and spring stiffness, not on anything else

- We can express the natural frequency in terms of Hz: $f_n = \frac{1}{2\pi}\omega_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (because 2π radians represent a full rotation/cycle)
- The natural period period is then $\frac{1}{f_n} = 2\pi\sqrt{\frac{m}{k}}$, with units of seconds
- The amplitude A and phase shift ϕ can be determined using the initial conditions (Initial Value Problem)

Adding Gravity

- If we introduce gravity, the equation becomes inhomogeneous: $ma(t) + kx(t) = 0 \implies mx''(t) + kx(t) = mg$
- The modified solution has form $x(t) = A\sin(\omega_n t + \phi) + \Delta_0$, where $k\Delta_0 = mg \implies \Delta_0 = \frac{mg}{k}$
  - Essentially the system will now oscillate around this shifted point instead, but other properties remain the same
  - The general solution to an inhomogeneous linear ODE is the general solution to the homogeneous DE plus a particular solution; intuitively in this case $x = \Delta_0$ represents the particular solution, where the force of the spring and gravity are in balance and no initial movement, so the system stays in that equilibrium forever

Other Methods for Calculating ω

- ω_n and f_n are important properties of the structure as it allows us to determine whether the structure is susceptible to time-varying loads (resonance)
- Measuring k can be difficult, so we can compute f_n from the static displacement Δ_0 instead:
  - $k\Delta_0 = mg \implies k = \frac{mg}{\Delta_0}$
  - $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{mg}{\Delta_0} \cdot \frac{1}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta_0}} \approx \frac{15.76}{\sqrt{\Delta_0}}\text{Hz}$
  - Note: This assumes Δ_0 has units of mm