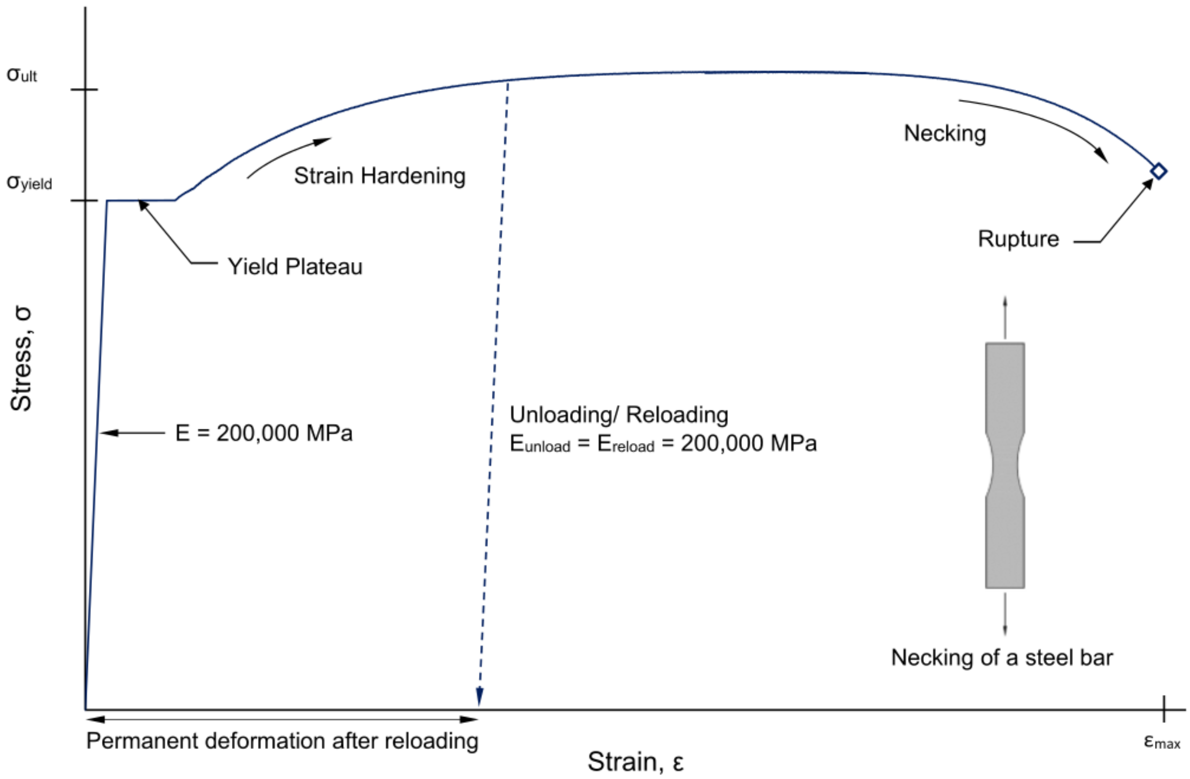
Lecture 6, Sep 21, 2021

Stress-Strain Curves



(Note: Mild steel is also sometimes known as low-carbon or low-alloy steel; the lower carbon content makes it more ductile and easier to work with, but less strong)

- Materials may exhibit linear elasticity for lower loads, but as they approach failure the stress-strain relationship is nonlinear
- Some key *material properties* are used to describe the stress-strain curve:
  1. Strength: How much stress a material can carry before it fails
     1. Yield strength/stress σ_y or f_y: The stress at which the material starts yielding (permanently deforming); when yielding occurs, strain will increase even without increasing stress; this is the *yield plateau* (flat part of graph)
        - The yield strain is typically about 0.002, and the yield plateau ends at about 0.05
     2. Ultimate strength/stress σ_u or f_u: The stress at which the material fails completely (the peak on the stress-strain curve)
  2. Ductility: How much a material can be deformed before it breaks; the largest strain a material can carry before it fractures (ductile vs brittle)
  3. Young's Modulus (material stiffness): Slope of the linear elastic region of the curve (see previous lecture)
     - Materials with higher value of E are *stiff*, lower values of E are *flexible*
- Plastic deformations are not recoverable; elastic deformations are
- Phases of the curve:
  1. The linear elastic region where $\sigma = E\varepsilon$ (small strains only)
  2. Yield plateau: Plastic behaviour; strain can change without stress changing (once the yield strength is reached)
  3. The rest of the graph, where the curve is nonlinear; some strengthening due to strain hardening and then softening as necking begins
- If the stress is unloaded when you're on the yield plateau, the stress-strain curve of unloading is linear

with the same slope as the original linear elastic region, but it won't pass through the origin
  - Essentially when unloaded the linear elastic region is shifted to the right by the same amount you moved along the yield plateau
- Strain hardening: When a material gets stronger and stiffer when strained beyond its yield point
- Necking: Local tensile strains cause the cross-sectional area to become much smaller; usually precedes failure
  - During this phase, the engineering stress goes down, but the true stress keeps going up because the cross-sectional area decreases
- Steel is very handy because it has a large yield plateau, so there is a lot of warning before it fails
- At the atomic level:
  - During the linear elastic region the atoms get pulled apart and they can spring back together
  - During the yield plateau, the atoms slip past each other
  - Past the yield plateau, the atoms get stuck so stress hardening happens

Strain Energy

- The energy stored in a material as it is deformed: $W = \int F \, \mathrm{d}\Delta l$
- During the linear elastic phase the strain energy is $\frac{1}{2}k(\Delta l)^2$
- The *strain energy density* $U = \int \sigma \, \mathrm{d}\varepsilon$ is the energy stored in the material per unit volume
  - $U = \frac{W}{V} = \int \frac{F}{V} \, \mathrm{d}\Delta l = \int \frac{F}{AL_0} \, \mathrm{d}\Delta l = \int \frac{F}{A} \, \mathrm{d}\frac{\Delta l}{L_0} = \int \sigma \, \mathrm{d}\varepsilon$
  - Also known as *specific strain energy*
  - Units of $\mathrm{MJ/m^3}$ since $\mathrm{MPa} = \mathrm{MN/m^2} = \mathrm{MN \cdot m/m^3} = \mathrm{MJ/m^3}$
  - $W = U \cdot V_0$ where V_0 is the original volume before deformation
  - In the linear elastic region: $W = \int \sigma \, \mathrm{d}\varepsilon \cdot V_0 = \int E\varepsilon \, \mathrm{d}\varepsilon \cdot V_0 = \frac{1}{2}E\varepsilon^2 V_0 = \frac{1}{2}\sigma\varepsilon V_0$
  - Strain energy density is a material property independent of the member
- Additional material properties:
  4. Resilience: Max amount of energy the material can absorb before yielding; the area under the curve in the linear elastic region
  5. Toughness: Max amount of energy the material can absorb before failing; the area under the entire stress-strain curve

Thermal Expansion

- Thermal strains ε_{th} are related to temperature changes by $\varepsilon_{th} = \alpha \Delta T$, where α is the *coefficient of thermal expansion*, a material constant
  - Thermal strains only cause stresses if the material is not allowed to expand/contract
  - Stresses caused by thermal strain can be calculated by Young's modulus if the length of the material is fixed